

Design a rectangular microstrip antenna to operate at a frequency of 1.5 GHz on a Rogers Corporation RO4003C substrate- assume $h = 0.813 \text{ mm} = 0.032''$, 1 oz. copper cladding ($34 \mu\text{m}$), loss tangent $\tan(\delta) = 0.0022$, and a relative dielectric constant $\epsilon_r = 3.38$. The antenna is to be matched to a 50Ω microstrip transmission line on this substrate. Discuss and justify design choices. Accurately sketch a top view of the final design.

EE 583 only- Include a fully-labeled Smith chart showing the normalized admittances $y_1 = y_2$ & y_{2t} (i.e., y_2 translated across length $L + \Delta L$ of microstrip antenna) and discuss results.

Design Procedure (All calculations done using to full precision of/on MathCad)

- 1) Specify ϵ_r and h of substrate, the desired resonant frequency f_r , and the impedance $Z_{c,\text{feed}}$ of the feeding transmission line.

$$\epsilon_r = 3.38, \quad h = 0.813 \text{ mm} = 0.032 \text{ in}, \quad f_r = 1.5 \text{ GHz}, \quad \& \quad Z_{c,\text{feed}} = 50 \Omega$$

- 2) Calculate width of patch using (14-6)

$$W = \frac{c}{2f_r} \sqrt{\frac{2}{\epsilon_r + 1}} = \frac{2.9979 \times 10^8}{2(1.5 \times 10^9)} \sqrt{\frac{2}{3.38 + 1}} \Rightarrow \underline{W = 67.527 \text{ mm}}$$

- 3) Calculate effective relative permittivity using (14-1)

$$\epsilon_{r,\text{eff}} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left[1 + 12 \frac{h}{W} \right]^{-0.5} = \frac{3.38 + 1}{2} + \frac{3.38 - 1}{2} \left[1 + 12 \frac{0.813}{67.527} \right]^{-0.5} \Rightarrow \underline{\epsilon_{r,\text{eff}} = 3.30236}$$

- 4) Calculate fringing length using (14-2)

$$\Delta L = 0.412 h \frac{(\epsilon_{r,\text{eff}} + 0.3) \left(\frac{W}{h} + 0.264 \right)}{(\epsilon_{r,\text{eff}} - 0.258) \left(\frac{W}{h} + 0.8 \right)} = 0.412(0.813) \frac{(3.3024 + 0.3) \left(\frac{67.527}{0.813} + 0.264 \right)}{(3.3024 - 0.258) \left(\frac{67.527}{0.813} + 0.8 \right)} \Rightarrow \underline{\Delta L = 0.3938 \text{ mm}}$$

- 5) Calculate the effective length (14-5) and guided wavelength λ .

$$L_{\text{eff}} = \frac{c}{2f_r \sqrt{\epsilon_{r,\text{eff}}}} = \frac{2.9979 \times 10^8}{2(1.5 \times 10^9) \sqrt{3.30236}} \Rightarrow \underline{L_{\text{eff}} = 54.99 \text{ mm}}$$

and

$$\lambda = 2L_{\text{eff}} = 2(54.99) \Rightarrow \underline{\lambda = 109.98 \text{ mm}}$$

6) Calculate the patch length L (14-7) and L/λ . Then, calculate and check that $W/L < 2$.

$$L = L_{\text{eff}} - 2\Delta L = 54.99 - 2(0.3938) \Rightarrow \underline{L = 54.203 \text{ mm}}$$

$$L/\lambda = 54.203/109.98 \Rightarrow \underline{L/\lambda = 0.4928}$$

$$W/L = 67.527/54.203 \Rightarrow \underline{W/L = 1.246 \text{ (should be OK as it's } < 2)}$$

7) Calculate $G_1 \approx G_2$ and $B_1 \approx B_2$.

$$G_{1,\text{est}} = \frac{W}{120\lambda_0} \left[1 - \frac{1}{24} (k_0 h)^2 \right] \quad \text{for } \frac{h}{\lambda_0} < \frac{1}{10} \quad (14-8a)$$

$$B_{1,\text{est}} = \frac{W}{120\lambda_0} \left[1 - 0.636 \ln(k_0 h) \right] \quad \text{for } \frac{h}{\lambda_0} < \frac{1}{10} \quad (14-8b)$$

where

$$\lambda_0 = c/f_r = 2.9979 \times 10^8 / 1.5 \times 10^9 \Rightarrow \lambda_0 = 199.862 \text{ mm},$$

$$h/\lambda_0 = 0.813/199.862 \Rightarrow h/\lambda_0 = 0.004068,$$

$$\text{and } k_0 = 2\pi/\lambda_0 = 2\pi/0.199862 \Rightarrow k_0 = 31.4377 \text{ rad/m},$$

$$G_{1,\text{est}} = \frac{67.527}{120(199.862)} \left[1 - \frac{1}{24} (34.4377 * 0.813 * 10^{-3})^2 \right] \Rightarrow \underline{G_{1,\text{est}} = 2.8155 \text{ mS}}$$

$$B_{1,\text{est}} = \frac{67.527}{120(199.862)} \left[1 - 0.636 \ln(34.4377 * 0.813 * 10^{-3}) \right] \Rightarrow \underline{B_{1,\text{est}} = 9.3817 \text{ mS}}$$

numerically integrate (notes & 14-12a)-

$$G_1 = \frac{1}{\pi \eta_0} \int_{\theta=0}^{\pi} \left[\frac{\sin\left(\frac{k_0 W}{2} \cos \theta\right)}{\cos \theta} \right]^2 \sin^3 \theta d\theta \Rightarrow \underline{G_1 = 1.1798 \text{ mS}}$$

$$= \frac{1}{\pi(376.73)} \int_{\theta=0}^{\pi} \left[\frac{\sin\left(\frac{34.438(0.06753)}{2} \cos \theta\right)}{\cos \theta} \right]^2 \sin^3 \theta d\theta$$

$$\text{(notes) } B_1 = \left(\frac{G_1}{G_{1,\text{est}}} \right) B_{1,\text{est}} = \left(\frac{1.1798}{2.8155} \right) 9.3817 \Rightarrow \underline{B_1 = 3.9313 \text{ mS}}$$

$$\text{Therefore, } Y_1 = Y_2 = G_1 + j B_1 \Rightarrow \underline{Y_1 = Y_2 = 1.1798 + j 3.9313 \text{ mS}}$$

- 8) Calculate the TL characteristic impedance $Z_{c,ant}$ (14-19a) and admittance $Y_{c,ant}$ for the rectangular microstrip antenna.

$$Z_{c,ant} = \begin{cases} \frac{60}{\sqrt{\epsilon_{r,eff}}} \ln \left[\frac{8h}{W} + \frac{W}{4h} \right] & \frac{W}{h} \leq 1 \\ \frac{\eta_0}{\sqrt{\epsilon_{r,eff}} \left[\frac{W}{h} + 1.393 + 0.667 \ln \left(\frac{W}{h} + 1.444 \right) \right]} & \frac{W}{h} > 1 \end{cases}$$

here $W/h = 67.527/0.813 = 83.059$ and

$$Z_{c,ant} = \frac{376.73}{\sqrt{3.30236} \left[83.059 + 1.393 + 0.667 \ln(83.059 + 1.444) \right]} \Rightarrow \underline{Z_{c,ant} = 2.3717 \Omega}$$

and

$$Y_{c,ant} = \frac{1}{Z_{c,ant}} = \frac{1}{2.3717} \Rightarrow \underline{Y_{c,ant} = 0.4216 S}$$

- 9) (EE 583) Use Smith chart or direct calculation to verify $\tilde{Y}_2 = \tilde{G}_2 + j\tilde{B}_2 \approx G_1 - jB_1 = Y_1^*$.

On Smith chart (next page), plot $y_2 = Y_2/Y_{c,ant} = (1.1798 + j 3.9313)10^{-3} (2.3717)$

$$\Rightarrow \underline{y_2 = 0.0028 + j0.0093 S/S.}$$

Move $(L+\Delta L)/\lambda = (54.203+0.3938)/109.98 = 0.4964$ WAVELENGTHS TOWARD GENERATOR on arc of constant $|\Gamma|$.

On Smith chart, read off $\Rightarrow \underline{y_{2t} = 0.0028 - j0.013 S/S.}$

We approximately meet the condition that-

$$\underline{y_{2t} = 0.0028 - j0.013 S/S \approx y_{1*} = 0.0028 - j0.0093 S/S} \quad (\sim 15\% \text{ error on susceptance})$$

- 10) Numerically calculate the mutual conductance G_{12} (14-18a) between the slots

$$\begin{aligned} G_{12} &= \frac{1}{\pi \eta_0} \int_{\theta=0}^{\pi} \left[\frac{\sin \left(\frac{k_0 W}{2} \cos \theta \right)}{\cos \theta} \right]^2 J_0(k_0 L \sin \theta) \sin^3 \theta d\theta \\ &= \frac{1}{\pi (376.73)} \int_{\theta=0}^{\pi} \left[\frac{\sin \left(\frac{31.44(0.0675)}{2} \cos \theta \right)}{\cos \theta} \right]^2 J_0(31.44(0.0542) \sin \theta) \sin^3 \theta d\theta \end{aligned}$$

$$\Rightarrow \underline{G_{12} = 0.5828 mS}$$

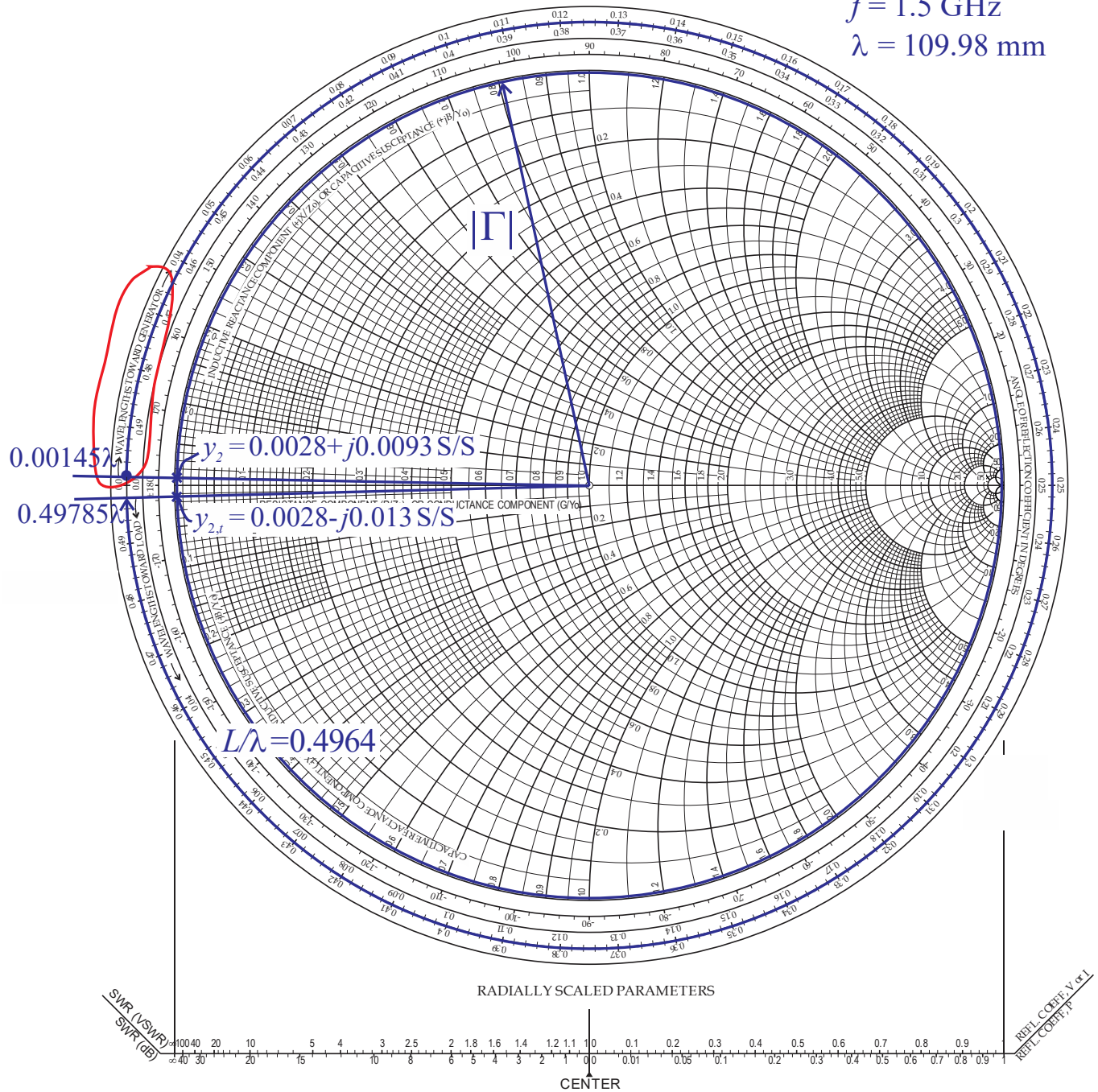
9) continued, EE 583 only

Simple Smith Chart

$$Z_{c,ant} = 2.3717 \Omega$$

$$f = 1.5 \text{ GHz}$$

$$\lambda = 109.98 \text{ mm}$$



11) Calculate R_{in} , use plus (+) sign in (14-17) equation.

$$Z_{in} = R_{in} = \frac{1}{2(G_1 + G_{12})} = \frac{1}{2(1.1798 + 0.5828)10^{-3}} \Rightarrow \underline{Z_{in} = R_{in} = 283.6752 \Omega}$$

- 12) Since an inset microstrip feed is required (i.e., $R_{in} \neq Z_{c,feed}$), calculate length y_0 of the inset needed to match the rectangular patch to the feeding transmission line. When $G_1/Y_{c,feed} \ll 1$ and $B_1/Y_{c,feed} \ll 1$, a good starting point is

$$y_0 \approx \frac{L}{\pi} \cos^{-1} \left(\sqrt{\frac{Z_{c,feed}}{R_{in}}} \right) \quad (14-20a).$$

Here, $G_1/Y_{c,feed} = 0.0011798/0.02 = 0.059 \ll 1$ (OK),

but $B_1/Y_{c,feed} = 0.00393/0.02 = 0.197 \ll 1$ (so-so).

$$\text{So, we will estimate } y_{0,est} \approx \frac{0.054203}{\pi} \cos^{-1} \left(\sqrt{\frac{50}{283.6752}} \right) \Rightarrow \underline{y_{0,est} = 19.63 \text{ mm}}$$

Check this estimate using (14-20a)-

$$\begin{aligned} R_{in}(y_0) &\approx \frac{1}{2(G_1 + G_{12})} \left[\cos^2 \left(\frac{\pi y_0}{L} \right) + \frac{G_1^2 + B_1^2}{Y_{c,feed}^2} \sin^2 \left(\frac{\pi y_0}{L} \right) - \frac{B_1}{Y_{c,feed}} \sin \left(\frac{2\pi y_0}{L} \right) \right] \\ &\approx 283.675 \left[\cos^2 \left(\frac{\pi y_0}{0.0542} \right) + \frac{0.00118^2 + 0.00393^2}{0.02^2} \sin^2 \left(\frac{\pi y_0}{0.0542} \right) - \frac{0.00393}{0.02} \sin \left(\frac{2\pi y_0}{0.0542} \right) \right] \\ &\Rightarrow \underline{R_{in}(y_{0,est} = 19.63 \text{ mm}) = 17.348 \Omega} \end{aligned}$$

Since $R_{in}(y_{0,est})$ is too low, iteratively shorten inset length y_0 using above equation-

$$\Rightarrow \text{With } \underline{y_0 = 16.4786 \text{ mm}}, R_{in}(y_0) = 50 \Omega = Z_{c,feed}.$$

- 13) Determine the width W_0 of the feeding microstrip transmission line.

Method 2: Iteratively guess the width W_0 of the feeding transmission line using-

$$\epsilon_{r,eff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left[1 + 12 \frac{h}{W_0} \right]^{-0.5} \quad (14-1)$$

and (14-19b)

$$Z_{c,feed} = \frac{\eta_0}{\sqrt{\epsilon_{r,eff}} \left[\frac{W_0}{h} + 1.393 + 0.667 \ln \left(\frac{W_0}{h} + 1.444 \right) \right]} \quad \frac{W_0}{h} > 1.$$

to get $Z_{c,feed} = 50 \Omega$ and $\epsilon_{r,eff} = 2.67$, when $W_0 = 2.331895 h = 1.8958 \text{ mm}$.

Using the Rogers Corporation Microwave Impedance Calculator (MWI 2018), $W_0 = 1.883 \text{ mm}$. Assume Rogers know their boards best and choose $\Rightarrow \underline{W_0 = 1.88 \text{ mm}}$.

14) Select the notch width n in the range $0.2W_0 < n < 0.5W_0$ (notes).

Arbitrarily select $n \approx 0.48W_0 \Rightarrow \underline{n = 0.9 \text{ mm}}$.

15) Draw the resulting design.

