6.27 Design a 19-element uniform linear scanning array with a spacing of $\lambda/4$ between the elements.

(a) What is the progressive phase excitation between the elements so that the maximum of the array factor is 30° from the line where the elements are placed?

(b) What is the half-power beamwidth (in degrees) of the array factor of part a?

(c) What is the value (in dB) of the maximum of the first minor lobe?

Verify using the computer program **Arrays** of this chapter.

- For parts b) & c), verify or find exact answers using Matlab or Mathcad or Arrays.

a) Per (6-21), $\beta = -kd\cos\theta_0$

$\beta = -\frac{2\pi}{\lambda}\frac{\lambda}{4}\cos 30^\circ \Rightarrow \underline{\beta = -1.36035\ \text{rad} = -77.942^\circ}$

b) Per (6-22), the HPBW is estimated as

$\theta_h = HPBW_{est} = \cos^{-1}\left[\cos\theta_0 - \frac{2.782}{Nkd}\right] - \cos^{-1}\left[\cos\theta_0 + \frac{2.782}{Nkd}\right]$

$HPBW_{est} = \cos^{-1}\left[\cos 30^\circ - \frac{2.782}{19(\pi/2)}\right] - \cos^{-1}\left[\cos 30^\circ + \frac{2.782}{19(\pi/2)}\right]$

$HPBW_{est} = 39.393^\circ - 16.415^\circ = \underline{22.978^\circ}$

Using MathCad, $\underline{HPBW = 23.02169^\circ}$
(See attached)

c) Per (6-17a), the maximum of the first sidelobe is down −13.46 dB from the $AF_{max.} = N = 19$ (see (6-10a)). Since $U \propto |AF|^2$,

$AF_{max} = 20\log_{10} 19 = 25.575$ dB.

First sidelobe max $\simeq 25.575 - 13.46 = \underline{12.115\ dB}$

Using MathCad, First sidelobe max $= \underline{12.395\ dB}$

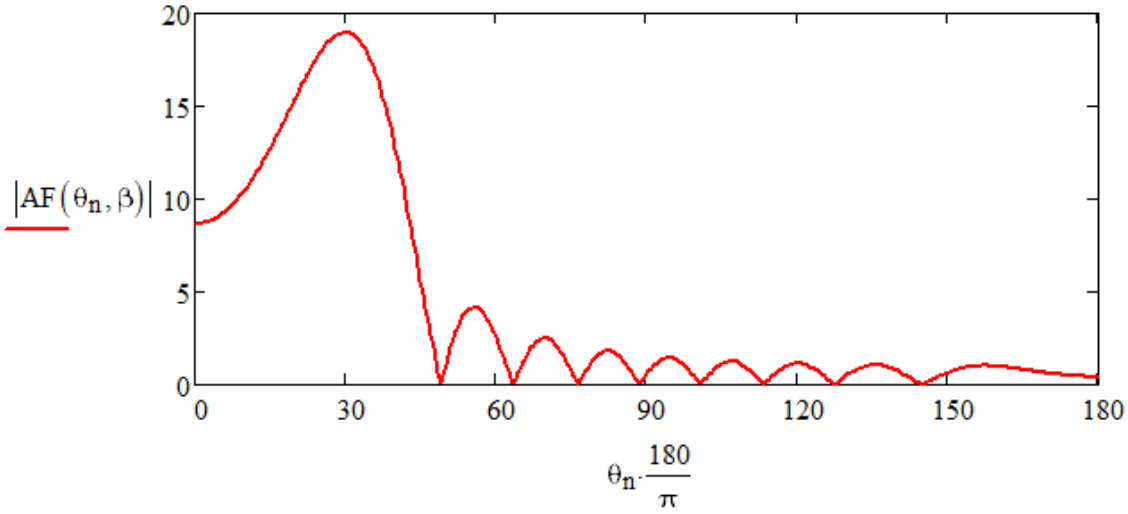
$\theta 0 := 30 \cdot \frac{\pi}{180}$ $N := 19$ $d\lambda := 0.25$ $kd := 2 \cdot \pi \cdot d\lambda$ $kd = 1.571$ rad

Per (6-21) $\beta := -kd \cdot \cos(\theta 0)$ $\boxed{\beta = -1.36035}$ rad

$\psi(\theta, \beta) := kd \cdot \cos(\theta) + \beta$ $n := 0 .. 360$ $\theta_n := \frac{\pi}{360} \cdot n - 0.0001$

$$AF(\theta, \beta) := \frac{\sin\left(\frac{N}{2} \cdot \psi(\theta, \beta)\right)}{\sin\left(\frac{1}{2} \cdot \psi(\theta, \beta)\right)}$$

Plot normalized array factor in rectangular format to get visual confirmation.



b) Find estimated & exact HPBW for array factor AF

Per (6-22) $\Theta h := \operatorname{acos}\left(\cos(\theta 0) - \frac{2.782}{N \cdot kd}\right) - \operatorname{acos}\left(\cos(\theta 0) + \frac{2.782}{N \cdot kd}\right)$

$\boxed{\Theta h \cdot \frac{180}{\pi} = 22.97801}$ deg

$\frac{19}{\sqrt{2}} = 13.43503$ $\theta h1 := 16.38478 \cdot \frac{\pi}{180}$ $\theta h2 := 39.40647 \cdot \frac{\pi}{180}$

$AF(\theta h1, \beta) = 13.43503$ $AF(\theta h2, \beta) = 13.43503$

$HPBW := (\theta h2 - \theta h1) \cdot \frac{180}{\pi}$ $\boxed{HPBW = 23.02169}$ deg

c) Find angle and value of maximum of first sidelobe. Compare with maximum of AF.

$\theta FSL := 55.62298 \cdot \frac{\pi}{180}$ $|AF(\theta FSL, \beta)| = 4.166202257$

$AF\_FSL\_dB := 20 \cdot \log(|AF(\theta FSL, \beta)|)$ $\boxed{AF\_FSL\_dB = 12.3948}$ dB

Note: $20 \cdot \log(|AF(\theta FSL, \beta)|) - 20 \log(N) = -13.1803$ dB

The text says that the FSL maximum is down from the AF maximum by -13.46 dB.