6.17 Ten isotropic elements are placed along the z-axis. Design a Hansen-Woodyard end-fire array with the maximum directed toward $\theta_0 = 0^\circ$. Find the:

(a) desired spacing
(b) progressive phase shift β (in radians)
(c) location of all the nulls (in degrees)
(d) first-null beamwidth (in degrees)
(e) directivity; verify using the computer program **Directivity** of Chapter 2.

- In part e), can verify using Matlab or MathCad.

a) Per (6-25), $d = \left(\frac{N-1}{N}\right)\frac{\lambda}{4} = \left(\frac{10-1}{10}\right)\lambda/4 \Rightarrow \underline{\underline{d = 0.225\lambda}}$

b) Per (6-23a), $\beta = -\left(kd + \frac{2.92}{N}\right) = -\left(\frac{2\pi}{\lambda}0.225\lambda + \frac{2.92}{10}\right)$

$\theta_{mb} = 0^\circ$

$\underline{\underline{\beta = -1.7057 \text{ rad}}} = -97.73^\circ$

c) Per (6-11), $\theta_n = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2n\pi}{N}\right)\right]$ $n = 1, 2, \ldots$ $n \neq N, 2N, \ldots$

$\theta_n = \cos^{-1}\left[\frac{\lambda}{2\pi 0.225\lambda}\left(+1.7057 \pm \frac{2n\pi}{10}\right)\right] = \cos^{-1}\left[\frac{1}{0.45\pi}\left(1.7057 \pm 0.2\pi n\right)\right]$

$n=1$ $\theta_n = \cos^{-1}(\cancel{1.651})$ or $\cos^{-1}(0.7621) = \underline{\underline{40.35^\circ}}$

$n=2$ $\theta_n = \cos^{-1}(\cancel{2.095})$ or $\cos^{-1}(0.31765) = \underline{\underline{71.479^\circ}}$

$n=3$ $\theta_n = \cos^{-1}(\cancel{2.54})$ or $\cos^{-1}(-0.1268) = \underline{\underline{97.284^\circ}}$

$n=4$ $\theta_n = \cos^{-1}(\cancel{2.98})$ or $\cos^{-1}(-0.5712) = \underline{\underline{124.836^\circ}}$

$n=5$ $\theta_n = \cos^{-1}(\cancel{3.429})$ or $\cos^{-1}(\cancel{1.106})$ No more nulls

d) $FNBW = 2|0^\circ - 40.35^\circ| = \underline{\underline{80.70^\circ}}$

For comparison, Table 6.5 approximate FNBW

$FNBW \simeq 2\cos^{-1}\left(1 - \frac{\lambda}{2dN}\right) \simeq 2\cos^{-1}(0.7) = 77.88^\circ$

Summarize information from 6.17
for 10 elements, 0 deg mainbeam

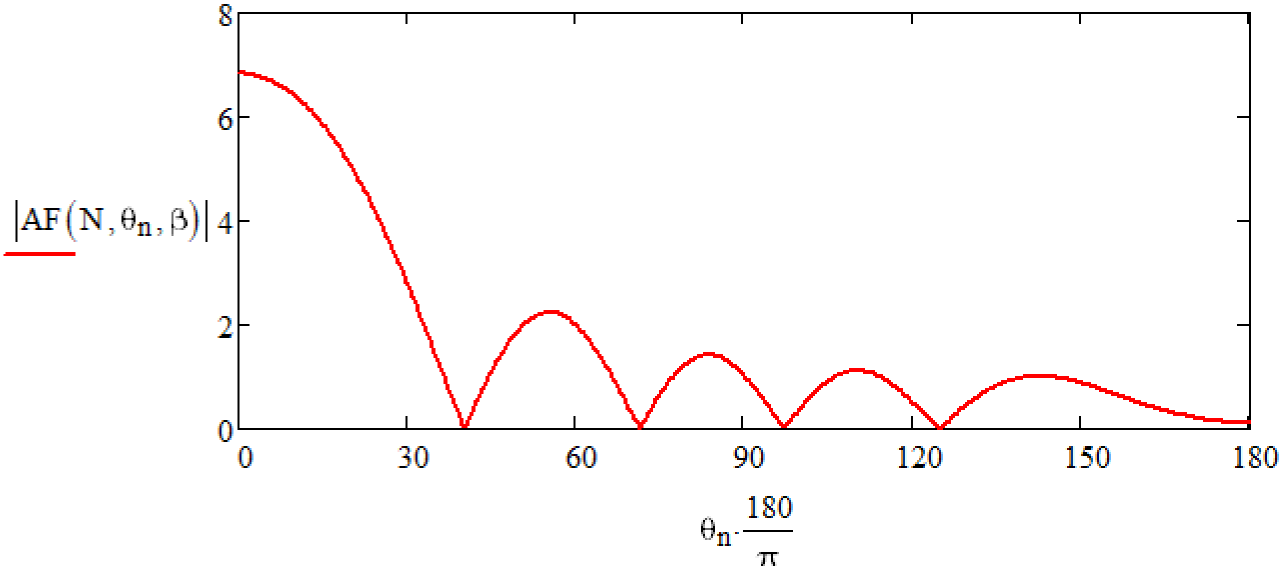
$N := 10 \qquad d\lambda := \frac{N-1}{N}\cdot 0.25$

$d\lambda = 0.225 \qquad kd := 2\cdot\pi\cdot d\lambda \qquad \beta := -\left(kd + \frac{2.92}{N}\right) \qquad \boxed{\beta = -1.705717}$ rad

$\psi(\theta,\beta) := kd\cdot\cos(\theta) + \beta \qquad n := 0..360 \qquad \theta_n := \frac{\pi}{360}\cdot n - 0.0001$

$$AF(N,\theta,\beta) := \frac{\sin\left(\frac{N}{2}\cdot\psi(\theta,\beta)\right)}{\sin\left(\frac{1}{2}\cdot\psi(\theta,\beta)\right)}$$

Plot normalized array factor in rectangular format to get visual confirmation of answers to parts c) & d).



$AF\left(N, 40.350\cdot\frac{\pi}{180}, \beta\right) = 6.97\times 10^{-6} \qquad AF\left(N, 97.2839\cdot\frac{\pi}{180}, \beta\right) = -2.61\times 10^{-6}$

$AF\left(N, 71.4786\cdot\frac{\pi}{180}, \beta\right) = 3.11\times 10^{-7} \qquad AF\left(N, 124.836\cdot\frac{\pi}{180}, \beta\right) = -4.69\times 10^{-6}$

$FNBW := 2\cdot(0 - 40.35) \qquad \boxed{FNBW = -80.7}$ deg

e) Calculate estimated maximum directivity using equation in Table 6.8 for a Hansen-Woodyard end-fire array with one maxima.

$Dest := 1.805\cdot 4\cdot N\cdot d\lambda \qquad Dest\_dB := 10\cdot\log(Dest)$

$\boxed{Dest = 16.245} \qquad \boxed{Dest\_dB = 12.107}$ dBi

e) cont.

Start computing exact directivity by calculating power radiated by the array P_{rad}.

Remember that the $|AF|^2$ is proportional to the radiation intensity

$$\mathrm{Prad} := 2\cdot\pi\cdot\left[\int_0^{\pi}\left[\frac{\sin\left[\frac{N}{2}\cdot(kd\cdot\cos(\theta)+\beta)\right]}{\sin\left[\frac{1}{2}\cdot(kd\cdot\cos(\theta)+\beta)\right]}\right]^2\cdot\sin(\theta)\,d\theta\right] \qquad \boxed{\mathrm{Prad} = 35.6735} \quad \text{'W'}$$

The maximum directivity $D_0 = 4\pi U_{max}/P_{rad} = 4\pi|AF_{max}|^2/P_{rad}$.

$$\mathrm{D0} := \frac{4\cdot\pi\cdot AF(N,0,\beta)^2}{\mathrm{Prad}} \qquad \mathrm{D0\_dB} := 10\cdot\log(\mathrm{D0})$$

Exact $\boxed{\mathrm{D0} = 16.44011}$ $\boxed{\mathrm{D0\_dB} = 12.159}$ dBi

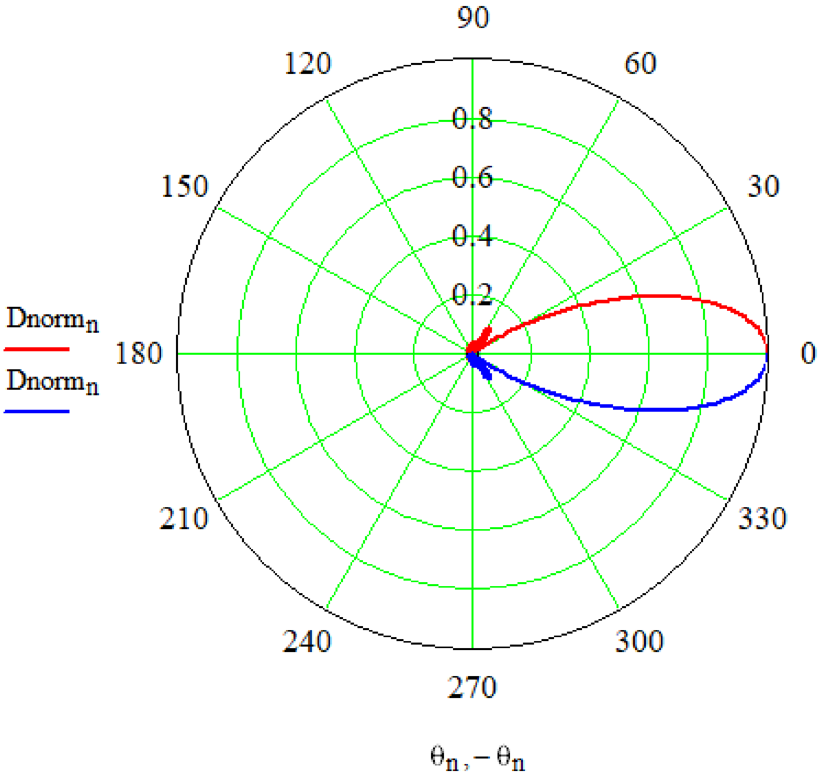
Pretty close to estimates. $\boxed{\mathrm{Dest} = 16.245}$ $\boxed{\mathrm{Dest\_dB} = 12.107}$ dBi

Bonus: plot directivity radiation pattern.

$$D_n := \frac{4\cdot\pi\cdot\left[\frac{\sin\left[\frac{N}{2}\cdot\left(kd\cdot\cos\left(\theta_n\right)+\beta\right)\right]}{\sin\left[\frac{1}{2}\cdot\left(kd\cdot\cos\left(\theta_n\right)+\beta\right)\right]}\right]^2}{\mathrm{Prad}} \qquad \mathrm{Dnorm}_n := \frac{D_n}{\mathrm{D0}}$$

$$\mathrm{D\_dB}_n := \mathrm{if}\left(\mathrm{Dnorm}_n < 0.0001, 0, 10\cdot\log\left(\mathrm{Dnorm}_n\right)+40\right)$$

Normalized Directivity (D0=16.44), N=10



Normalized Directivity (D0=12.16 dBi), N=10