6.13 Design an ordinary end-fire uniform linear array with only one maximum so that its directivity is **12** dB (above isotropic). The spacing between the elements is $\lambda/3$, and its length is much greater than the spacing. Determine the

(a) number of elements

(b) overall length of the array (in wavelengths)

(c) approximate half-power beamwidth (in degrees)

(d) amplitude level (compared to the maximum of the major lobe) of the first minor lobe (in dB)

(e) progressive phase shift between the elements (in degrees).

- For part a) choose the closest integer number of elements. In parts c)-d), plot AF over visible region of ψ using Matlab or MathCad and verify/find exact answers .

a) Per (6-49), $D_0 \simeq 4N(d/\lambda)$

$D_0 = 10^{12/10} = 15.849 = 4N(1/3) \Rightarrow N = 11.99 \rightarrow \underline{\underline{N=12}}$

b) Per (6-43), $L = (N-1)d = (12-1)\lambda/3 \Rightarrow \underline{\underline{L = 3.\overline{66}\lambda}}$

c) From Table 6.4

$$HPBW = \theta_h \simeq 2\cos^{-1}\left(1 - \frac{1.391\lambda}{\pi d N}\right) = 2\cos^{-1}\left(1 - \frac{1.391\lambda}{\pi \frac{1}{3} 12}\right)$$

$$\underline{\underline{HPBW = \theta_h \simeq 54.427^\circ}}$$

From MathCad, $\underline{\underline{HPBW = 54.5218^\circ}}$

d) From (6-17a), $\underline{\underline{AF_{FML} = -13.46\,dB}}$

From MathCad, $\underline{\underline{AF_{FML} = -13.057\,dB}}$

e) For $\theta_{mB} = 0^\circ$, (6-20a) $\underline{\underline{\beta_{0^\circ} = -kd = \frac{-2\pi}{\lambda}\lambda/3 = -\frac{2\pi}{3} = -120^\circ}}$

$\theta_{mB} = 180^\circ$, (6-20b) $\underline{\underline{\beta_{180^\circ} = kd = \frac{2\pi}{3} = 120^\circ}}$

$d\lambda := \frac{1}{3}$ $kd := 2\cdot\pi\cdot d\lambda$ $N := 12$ $I0 := 1$ $\beta := -kd$

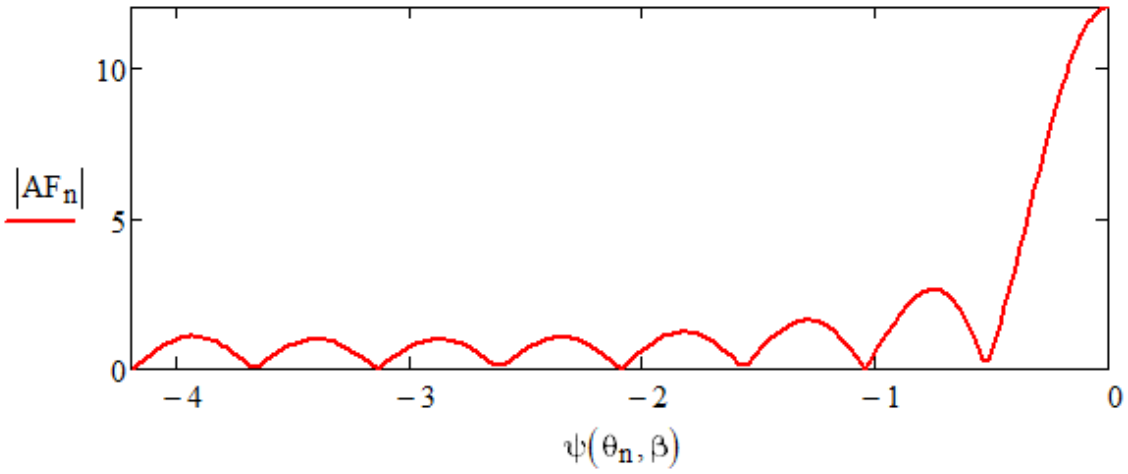
$\psi(\theta,\beta) := kd\cdot\cos(\theta) + \beta$ $n := 0..180$ $\theta_n := \frac{\pi}{180}\cdot n - 0.0001$

$AF_n := I0\cdot\frac{\sin\left(\frac{N}{2}\cdot\psi(\theta_n,\beta)\right)}{\sin\left(\frac{1}{2}\cdot\psi(\theta_n,\beta)\right)}$

$\beta = -2.094$ rad

$\beta\cdot\frac{180}{\pi} = -120$ deg

AF with d = lambda/3



Half-power point $\theta h := 27.2609\cdot\frac{\pi}{180}$ $\left(\left|\frac{\sin\left(\frac{N}{2}\cdot\psi(\theta h,\beta)\right)}{12\cdot\sin\left(\frac{1}{2}\cdot\psi(\theta h,\beta)\right)}\right|\right)^2 = 0.5$

$HPBWdeg := 2\cdot\theta h\cdot\frac{180}{\pi}$ $HPBWdeg = 54.5218$ deg

First sidelobe max $\theta 1SL := 50.09\cdot\frac{\pi}{180}$ $\psi(\theta 1SL,\beta) = -0.75067$

$\left|\frac{\sin\left(\frac{N}{2}\cdot\psi(\theta 1SL,\beta)\right)}{\sin\left(\frac{1}{2}\cdot\psi(\theta 1SL,\beta)\right)}\right| = 2.66888082$

$FSLmax := 20\cdot\log\left(\left|\frac{\sin\left(\frac{N}{2}\cdot\psi(\theta 1SL,\beta)\right)}{12\cdot\sin\left(\frac{1}{2}\cdot\psi(\theta 1SL,\beta)\right)}\right|\right)$ $FSLmax = -13.057$ dB