For the array in 6.3, find: a) location(s) of the maxima θ_m, b) the power radiated by the array P_{rad}, c) a function for the directivity D & D_{max}, and d) the half-power points θ_h and half-power beamwidth HPBW (analytically or determine numerically using MATLAB, MathCad, …). Plot the normalized directivity polar radiation pattern (in dB) in the x-z plane. Use a 0 to -40 dB scale with $\theta = 0$ at the top.

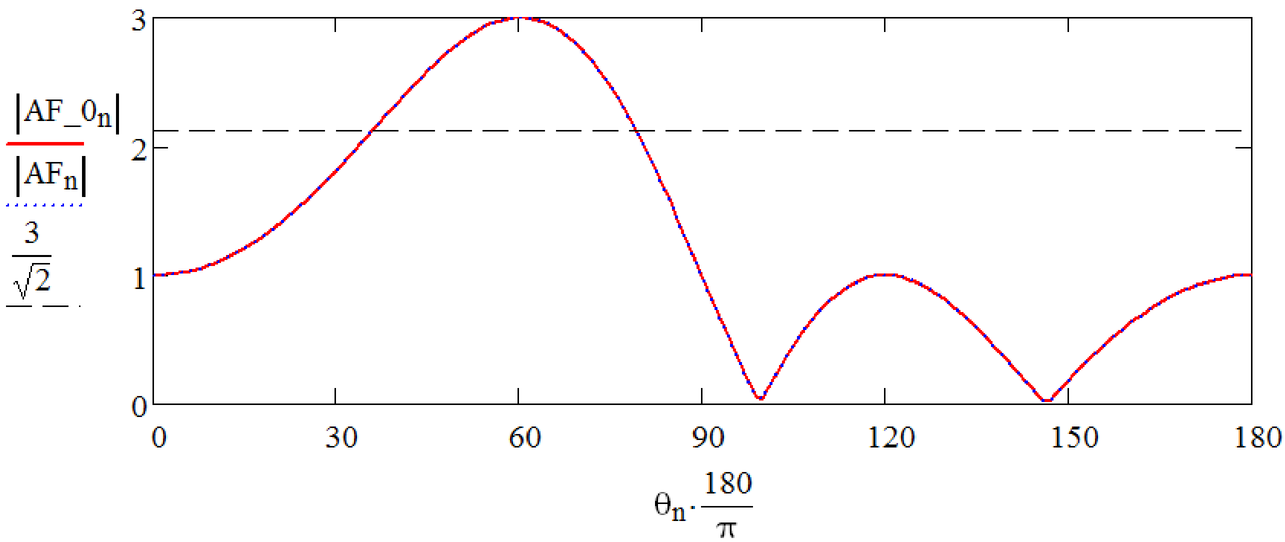
Using MathCad

Summarize information from 6.3 and plot AF (in rectangular format) as visual aid

$d\lambda := 0.5 \qquad kd := 2\cdot\pi\cdot d\lambda \qquad N := 3 \qquad I0 := 1 \qquad \beta := \frac{-\pi}{2}$

$\psi(\theta,\beta) := kd\cdot\cos(\theta) + \beta \qquad n := 0..180 \qquad \theta_n := \frac{\pi}{180}\cdot n - 0.0001$

$$AF\_0_n := I0\cdot\frac{\sin\left(\frac{N}{2}\cdot\psi(\theta_n,\beta)\right)}{\sin\left(\frac{1}{2}\cdot\psi(\theta_n,\beta)\right)} \qquad AF_n := 1 + 2\cdot\sin(\pi\cdot\cos(\theta_n))$$



a) location(s) of the maxima θ_m

The maxima θ_m occur when $AF_{max} = 1 + 2\sin(\pi\cos(\theta_m)) = N = 3$, solving for θ_m yields

$$\sin(\pi\cos(\theta_m)) = 1$$

which implies that

$$\pi\cos(\theta_m) = \pi/2$$

and

$$\cos(\theta_m) = 1/2$$

So, as we can see in above graph, we get $\boldsymbol{\theta_m = \cos^{-1}(1/2) =}$ **<u>60 degrees</u>**

b) the power radiated by the array P_{rad}

Remember that the AF squared is proportional to the radiation intensity

$$\mathrm{Prad} := 2\cdot\pi\cdot\left[\int_0^{\pi}\left[\mathrm{I0}\cdot\frac{\sin\left[\frac{N}{2}\cdot(kd\cdot\cos(\theta)+\beta)\right]}{\sin\left[\frac{1}{2}\cdot(kd\cdot\cos(\theta)+\beta)\right]}\right]^2\cdot\sin(\theta)\,d\theta\right] \qquad \boxed{\mathrm{Prad} = 37.699} \ \mathrm{W}$$

OR

$$\mathrm{Prad2} := 2\cdot\pi\cdot\left[\int_0^{\pi}(1+2\cdot\sin(\pi\cdot\cos(\theta)))^2\cdot\sin(\theta)\,d\theta\right] \qquad \mathrm{Prad2} = 37.699 \ \mathrm{W}$$

Note: $P_{rad} = \quad 12\pi = 37.699 \quad \mathrm{W}$

c) an expression for the directivity D & D_{max}

Again, remember that the AF squared is proportional to the radiation intensity

$$D = 4\pi U / P_{rad} = 4\pi \mathrm{AF}^2 / P_{rad}$$

So

$$D = 4\pi\{\sin[1.5\pi\cos(\theta) - 0.75\pi]/\sin[0.5\pi\cos(\theta) - 0.25\pi]\}^2 / 37.699$$

$$\underline{\boldsymbol{D = \{\sin[1.5\pi\cos(\theta) - 0.75\pi]/\sin[0.5\pi\cos(\theta) - 0.25\pi]\}^2 / 3}}$$

OR

$$D = 4\pi[1 + 2\sin(\pi\cos(\theta))]^2 / 37.699$$

$$\underline{\boldsymbol{D = [1 + 2\sin(\pi\cos(\theta))]^2 / 3}}$$

Note that $D_{max} = 4\pi \mathrm{AF}_{max}{}^2 / P_{rad} = 4\pi N^2 / P_{rad} = 4\pi(3)^2 / 12\pi$

$$\underline{\boldsymbol{D_{max} = 3 = 10\log(3) = 4.77\ \mathrm{dBi}}}.$$

d) the half-power points θ_h and half-power beamwidth HPBW (analytically or determine numerically using MATLAB, MathCad, …)

Again, remembering that the |AF| is proportional to |E| and that AFmax = 3. Therefore, we find the angles at which $(|AF|/AFmax)^2 = 0.5$

$$\theta h1 := 35.85055 \quad \text{deg}$$

$$\left(\frac{\left|1 + 2\cdot\sin\left(\pi\cdot\cos\left(\theta h1\cdot\frac{\pi}{180}\right)\right)\right|}{3}\right)^2 = 0.5$$

$$\theta h2 := 79.07915 \quad \text{deg}$$

$$\left(\frac{\left|1 + 2\cdot\sin\left(\pi\cdot\cos\left(\theta h2\cdot\frac{\pi}{180}\right)\right)\right|}{3}\right)^2 = 0.5$$

$$HPBW := \theta h2 - \theta h1$$

$$HPBW = 43.229 \quad \text{deg}$$

These results agree with the AF plot shown on the first page.

Plot the normalized <u>directivity</u> polar radiation pattern (in dB) in the *x-z* plane. Use a 0 to -40 dB scale with $\theta = 0$ at the top.

Use MATLAB and the radpat.m m-file for radiation pattern plot

```
% EE 483 problem 6.3 supplement (p6_03_supplement.m)
% Plot elevation pattern (wrt theta) for
% D = [1 + 2sin(pi*cos(theta))]^2 / 3
%
theta = -pi : pi/180 : pi;  % vary elevation angle
D = [1 + 2.*sin(pi*cos(theta))].^2/3;
% ******* Plot U in dB format *******
radpat(theta*180/pi,abs(D),'r-')
title('Prob 6.3 Normalized Directivity(dB)','fontsize',16,'fontname','times roman'),
%
set(findobj('type','line'),'linewidth',1.5)
set(findobj('type','line'),'markersize',14)
set(findobj('type','axes'),'linewidth',2)
```

From the MATLAB command window

Are input values in dB (Y/N)[Y]? N
Input values proportional to power (Y/N) [Y]? Y
Normalize to the Maximum Gain Value (Y/N)[Y]? Y
Minimum dB value at plot center [-40]? -40
Are the angles theta values? (Y/N)[Y]? Y
Labels on Vertical or Horizontal axis (V/H)[V]? V
Pattern line width [1.25]: 2
Line type of grid(-, --, -., :)[:]? :
>>