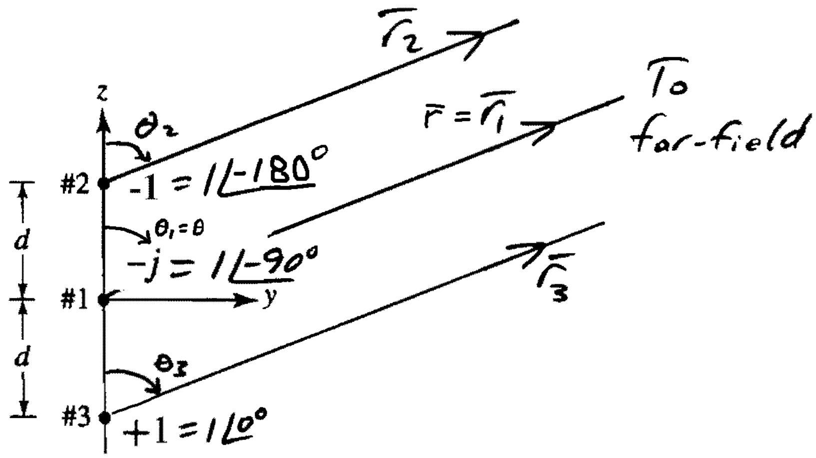
6.3 A three-element array of isotropic sources has the phase and magnitude relationships shown. The spacing between the elements is $d = \lambda/2$.

(a) Find the array factor.

(b) Find all the nulls.



a) <u>Method 1</u> (Hard way)

Put isotropic excitations in polar form. Note that $|I_0| = 1$ and the progressive phase $\beta = -90° = -\pi/2$. Assume an isotropic source at the origin has a phasor vector electric field $\bar{E}_{iso} = \hat{a}\,\dfrac{C_0 I_0 e^{-jkr}}{4\pi r}$ where C_0 is a constant.

$$\bar{E}_{Tot} = \bar{E}_1 + \bar{E}_2 + \bar{E}_3$$

Following the approximations of (6-2), let

$\theta = \theta_1 \simeq \theta_2 \simeq \theta_3$

$\bar{r}_1 = \bar{r} \Rightarrow$ both phase + magnitude

For amplitude, $|\bar{r}_2| \simeq |\bar{r}_3| = r$

For phase, $|\bar{r}_2| = r - d\cos\theta$

$|\bar{r}_3| = r + d\cos\theta$

a) cont.

$$\bar{E}_{tot} = \hat{a}\,\frac{C_o|I_o'|\,e^{-j\pi/2}\,e^{-jkr}}{4\pi r} + \hat{a}\,\frac{C_o|I_o'|\,e^{-j\pi}\,e^{-jk(r-\frac{\lambda}{2}\cos\theta)}}{4\pi r}$$

$$+\,\hat{a}\,\frac{C_o|I_o'|\,e^{j0}\,e^{-jk(r+\frac{\lambda}{2}\cos\theta)}}{4\pi r}$$

$$=\hat{a}\,\frac{C_o e^{-j\pi/2}e^{-jkr}}{4\pi r}\underbrace{\left[1+e^{-j\pi/2}\,e^{+jk\frac{\lambda}{2}\cos\theta}+e^{+j\pi/2}\,e^{-jk\frac{\lambda}{2}\cos\theta}\right]}_{\text{Array Factor (AF)}}$$

Now, use Euler's identity, $e^{\pm j\pi/2} = \pm j$, to write

$$AF = 1 - j\,e^{jk\frac{\lambda}{2}\cos\theta} + j\,e^{-j\frac{\lambda}{2}\cos\theta}$$

Per trigonometric identity / Euler's, we know

$$\sin A = \frac{e^{jA}-e^{-jA}}{2j} \Rightarrow 2\sin A = -j\,e^{jA} + j\,e^{-jA}$$

So, $AF = 1 + 2\sin\left(k\frac{\lambda}{2}\cos\theta\right) = 1 + 2\sin\left(\frac{2\pi}{\lambda}\frac{\lambda}{2}\cos\theta\right)$

$$\underline{\underline{AF = 1 + 2\sin(\pi\cos\theta)}}$$

<u>Method 2</u> Note: $d = \frac{\lambda}{2}$ & $\beta = -90° = -\pi/2$

Per (6-7a), $\psi = kd\cos\theta + \beta = \frac{2\pi}{\lambda}\frac{\lambda}{2}\cos\theta - \pi/2$

$$= \pi\cos\theta - \pi/2$$

Per (6-10a), $AF = \dfrac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\psi/2\right)}$ where $N = 3$

$$\underline{\underline{AF = \frac{\sin\left(\frac{3}{2}\pi\cos\theta - 3\pi/4\right)}{\sin\left(\frac{\pi}{2}\cos\theta - \pi/4\right)}}}$$

b) Find the nulls

Method 1 $AF = 1 + 2\sin(\pi\cos\theta_n) = 0$

$\Rightarrow \sin(\pi\cos\theta_n) = -\tfrac{1}{2} = -0.5$

$\Rightarrow \pi\cos\theta_n = \sin^{-1}(-0.5) = -\pi/6 \text{ or } \frac{-5\pi}{6}$

$\Rightarrow \cos\theta_n = -1/6 \text{ or } -5/6$

$\Rightarrow \theta_n = \cos^{-1}(-1/6) \text{ or } \cos^{-1}(-5/6)$

$$\underline{\underline{\theta_n = 99.594^\circ \text{ and } 146.443^\circ}}$$

Method 2 Use (6-11)

$$\theta_n = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2n}{N}\pi\right)\right] \quad n = 1, 2, \ldots \quad n \neq N, 2N \ldots$$

where $d = \lambda/2$, $\beta = -\pi/2$, & $N = 3$

$$\theta_n = \cos^{-1}\left[\frac{1}{\pi}\left(+\pi/2 \pm \frac{2n}{3}\pi\right)\right] = \cos^{-1}\left[1/2 \pm \frac{2n}{3}\right]$$

$n = 1 \quad \theta_n = \cos^{-1}\left[1/2 \pm 2/3\right] = \cos^{-1}(-1/6) \text{ or } \cancel{\cos^{-1}(7/6)}$ NO

$= 99.594^\circ$

$n = 2 \quad \theta_n = \cos^{-1}\left[1/2 \pm 4/3\right] = \cos^{-1}(-5/6) \text{ or } \cancel{\cos^{-1}(11/6)}$ NO

$= 146.443^\circ$

$n > 2$ No real solutions

$\Downarrow$

$$\underline{\underline{\theta_n = 99.594^\circ \text{ and } 146.443^\circ}}$$