5.13 Design a lossless resonant circular loop operating at 10 MHz so that its single-turn radiation resistance is 0.73 ohms. The resonant loop is to be connected to a matched load through a balanced "twin-lead" ~~300~~ 50-ohm transmission line.

(a) Determine the radius of the loop (in meters and wavelengths).

(b) To minimize the matching reflections between the resonant loop and the ~~300~~ 50-ohm transmission line, determine the closest number of integer turns the loop must have.

(c) For the loop of part b, determine the maximum power that can be expected to be delivered to a receiver matched load if the incident wave is polarization matched to the lossless resonant loop. The power density of the incident wave is 10^{-6} watts/m^2.

- Change 300 Ω to 50 Ω for transmission line. For part (c), first calculate the maximum effective area (refer to Chapter 2). Add part (d) If the antenna is made of aluminum wire (20 AWG, 3.5×10^7 S/m) with a turn spacing of 1.14 mm, compute the loss resistance and efficiency. ['Resonant' means assume reactance is zero.]

a) Since $R_r = 0.73\,\Omega$, assume a small loop and use (5-24) $R_r = \eta(\pi/6)(ka)^4$

$$0.73 = 376.7303\,(\pi/6)\left(\tfrac{2\pi}{\lambda}a\right)^4 = 376.7303(\pi/6)(2\pi)^4(a/\lambda)^4$$

$\hookrightarrow (a/\lambda)^4 = 2.3745\times10^{-6} \Rightarrow a/\lambda = 0.039255$

$$@\ f = 10\text{ MHz},\quad \lambda = c/f = \frac{2.998\times10^8}{10\times10^6} = 29.98\text{ m}$$

$$\underline{\underline{a = 0.039255\lambda = 1.17686\text{ m}}}$$

b) Use (5-24) + (5-24a) $R_{r,multi} = N^2 R_{r,single} = Z_{TL}$

$$N^2(0.73) = 50$$

$$N^2 = \frac{50}{0.73} = 68.493$$

$$N = 8.276 \Rightarrow \underline{\underline{N = 8}}$$

$$R_{r,multi} = 8^2(0.73) = 46.72\,\Omega$$

c) Per (2-112), $A_{em} = e_{cd}(1-|\Gamma|^2)\left(\frac{\lambda^2}{4\pi}\right)D_0 |\hat{\rho}_w \cdot \hat{\rho}_a|^2$

lossless $\Rightarrow e_{cd} = 1$, $\lambda = 29.98$

$\Gamma = \frac{Z_{loop} - Z_{TL}}{Z_{loop} + Z_{TL}} = \frac{46.72-50}{46.72+50} = -0.033912324$

(5-31) $D_0 = 3/2 = 1.5$

polarization matched $\Rightarrow |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = 1$

$A_{em} = 1(1-0.0339^2)\left(\frac{29.98^2}{4\pi}\right)1.5(1)$

$\underline{\underline{A_{em} = 107.163 \text{ m}^2}}$

Per (2-94), $A_e = \frac{P_T}{W_i} \Rightarrow P_{T,max} = A_{em} W_i$

$P_{T,max} = 107.163(10^{-6}) \Rightarrow \underline{\underline{P_{T,max} = 107.163 \mu W}}$

d) Per (5-25), $R_{ohmic} = R_L = \frac{Na}{b} R_S \left(\frac{R_p}{R_0} + 1\right)$

Now, $\sigma = 3.5 \times 10^7$ S/m & $2c = 1.14$ mm

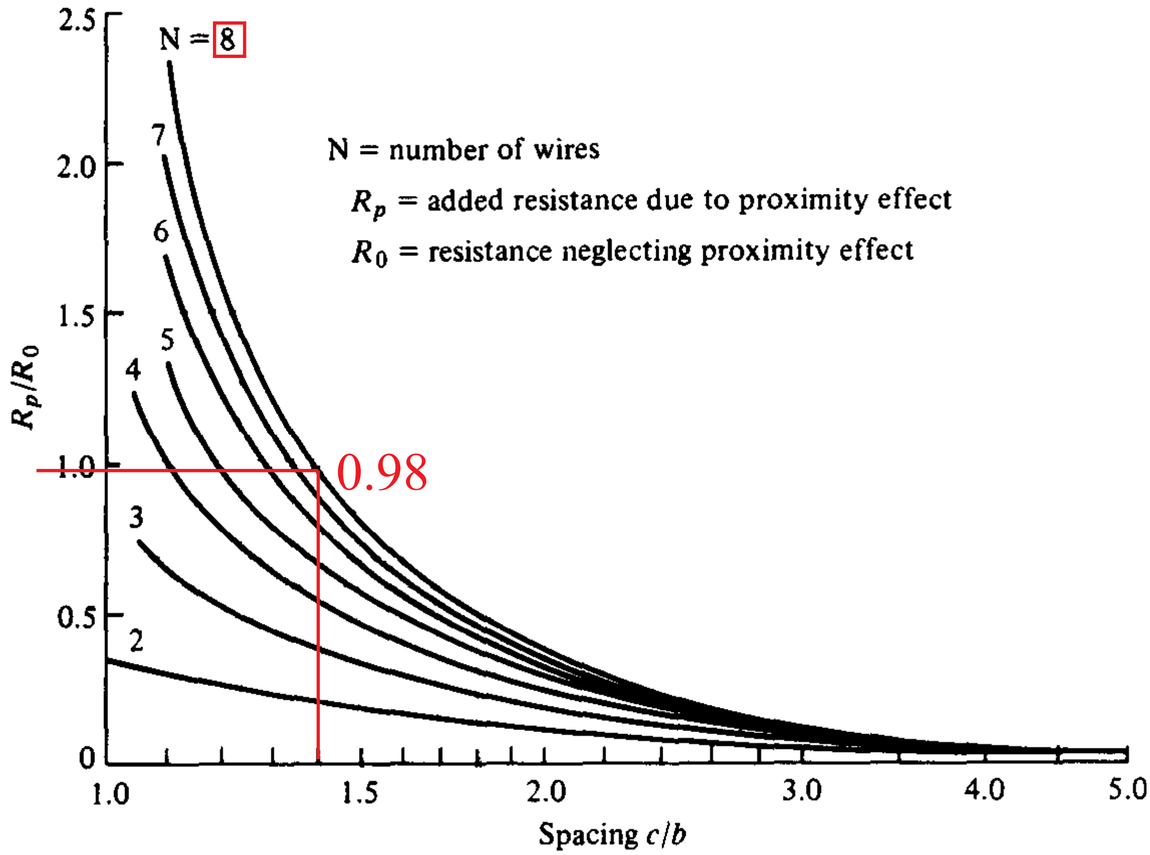
20 AWG $\Rightarrow d = 2b = 0.8128$ mm

$N = 8$, $a = 1.17686$ m, $b = \frac{0.8128}{2} = 0.4064$ mm

$R_S = \sqrt{\frac{\omega \mu_0}{2\sigma}} = \sqrt{\frac{2\pi(10\times10^6)4\pi\times10^{-7}}{2(3.5\times10^7)}} = 0.00106205 \ \Omega$

Using Fig 5.3 with $N = 8$ & $c/b = \frac{2c}{2b} = \frac{1.14}{0.8128} = 1.403$,

we read $R_p/R_0 = 0.98$



(b) Ohmic resistance due to proximity (after G. N. Smith)

Figure 5.3 *N*-turn circular loop and ohmic resistance due to proximity effect. (SOURCE: G. S. Smith, "Radiation Efficiency of Electrically Small Multiturn Loop Antennas," *IEEE Trans. Antennas Propagat.*, Vol. AP-20, No. 5, pp. 656-657, Sept. 1972© (1972) IEEE).

So, $R_L = \dfrac{8(1.17686)}{0.4064\times10^{-3}}(0.001062)(0.98+1)$

$R_L = 48.716\ \Omega$

Per (2-90), $e_{cd} = \dfrac{R_r}{R_r + R_L} = \dfrac{46.72}{46.72 + 48.716}$

$e_{cd} = 0.4895 = 48.95\ \%$