5.11 A single-turn resonant circular loop with a $\lambda/8\pi$ radius is made of copper wire with a wire radius of $10^{-4}\lambda/2\pi$ and conductivity of 5.7×10^7 S/m. For a frequency of 100 MHz, determine, *assuming uniform current*, the

(a) radiation efficiency (assume the wire is straight);

(b) *maximum gain* of the antenna (*dimensionless* and *in dB*).

- As part of (a), find the loss and radiation resistances.

a) For a small loop w/ constant current

(5-24) $R_r = \eta\left(\frac{\pi}{6}\right)(ka)^4$

$$R_r = 376.7303\left(\frac{\pi}{6}\right)\left(\frac{2\pi}{\lambda}\frac{\lambda}{8\pi}\right)^4 \Rightarrow \underline{\underline{R_r = 0.77053\ \Omega}}$$

Per (5-25) (2-90b) $R_{ohmic} = \frac{a}{b}\sqrt{\frac{\omega\mu_0}{2\sigma}} = \frac{\lambda/8\pi}{10^{-4}\lambda/2\pi}\sqrt{\frac{2\pi(100\times10^6)4\pi\times10^{-7}}{2(5.7\times10^7)}}$

$$\underline{\underline{R_{ohmic} = R_L = 6.57934\ \Omega}}$$

Per (2-90) $e_{cd} = \frac{R_r}{R_r + R_L} = \frac{0.7705}{6.579 + 0.7705}$

$$\underline{\underline{e_{cd} = 0.104836 = 10.4836\%}}$$

b) For a small loop w/ constant current, the directivity, per (5-31), is $D_0 = 3/2$

Per (2-49a), the gain is

$$G_0 = e_{cd} D_0 = (0.104836)\,3/2 = 10\log_{10}(0.15725)$$

$$\underline{\underline{G_0 = 0.15725 = -8.034\ dBi}}$$