5.3 A circular loop, *of loop radius* $\lambda/30$ *and wire radius* $\lambda/1000$, is used as a transmitting/receiving antenna in a back-pack radio communication system at *10 MHz*. The wire of the loop is made of copper with a conductivity of 5.7×10^7 S/m. Assuming the antenna is *radiating in free space*, determine the

(a) radiation resistance of the loop;

(b) loss resistance of the loop (*assume that its value is the same as if the wire were straight*);

(c) input resistance;

(d) input impedance;

(e) radiation efficiency.

➢ Change radius to $\lambda/35$. Hint: Read section 5.2.7 to find reactance.

a) Per (5-24), $R_r = \eta\left(\frac{\pi}{6}\right)(k^2a^2)^2 = 376.7303\left(\frac{\pi}{6}\right)\left(\frac{2\pi}{\lambda}\frac{\lambda}{35}\right)^4$ $\Rightarrow$ **$R_r = 0.20487\ \Omega$**.

b) Per (2-90b), $R_L = R_{hf} = \frac{\ell}{P}\sqrt{\frac{\omega\mu_0}{2\sigma}} = \frac{2\pi(\lambda/35)}{2\pi(\lambda/1000)}\sqrt{\frac{2\pi(10\times10^6)4\pi\times10^{-7}}{2(5.7\times10^7)}}$

$\Rightarrow$ **$R_L = 0.023778\ \Omega$**.

c) Per (5-33), $R_{in} = R_r + R_L = 0.20487 + 0.023778$ $\Rightarrow$ **$R_{in} = 0.22865\ \Omega$**.

d) Per section 5.2.7,

(5-37a), $L_A = \mu_0 a\left[\ln\left(\frac{8a}{b}\right) - 2\right] = 4\pi\times10^{-7}\left(\frac{2.9979\times10^8}{10\times10^6(35)}\right)\left[\ln\left(\frac{8(\lambda/35)}{\lambda/1000}\right) - 2\right]$

$\Rightarrow L_A = 3.69391674\times10^{-6}$ H.

Per (5-33), $X_A = \omega L_A = 2\pi(10\times10^6)\ 3.69391674\times10^{-6}$ $\Rightarrow X_A = 232.095634\ \Omega$.

(5-38), $L_i = \frac{a}{\omega b}\sqrt{\frac{\omega\mu_0}{2\sigma}} = \frac{\lambda/35}{2\pi(10\times10^6)(\lambda/1000)}\sqrt{\frac{2\pi(10\times10^6)4\pi\times10^{-7}}{2(5.7\times10^7)}}$

$\Rightarrow L_i = 3.784378\times10^{-10}$ H.

Per (5-33), $X_i = \omega L_i = 2\pi(10\times10^6)\ 3.784378\times10^{-10}$ $\Rightarrow X_i = 0.023778\ \Omega$.

$X_{in} = X_A + X_i = 232.095634 + 0.023778$ $\Rightarrow X_{in} = 232.11941\ \Omega$.

$Z_{in} = R_{in} + jX_{in}$ $\Rightarrow$ **$Z_{in} = 0.22865 + j\,232.11941\ \Omega$**.

e) Per (2-90), $e_{cd} = R_r/(R_r + R_L) = 0.20487 / 0.22865$ $\Rightarrow$ **$e_{cd} = 0.896 = 89.6\%$**