5.3 A circular loop, *of loop radius $\lambda/30$ and wire radius $\lambda/1000$*, is used as a transmitting/receiving antenna in a back-pack radio communication system at $20\,MHz$. The wire of the loop is made of copper with a conductivity of $5.7 \times 10^7\ S/m$. Assuming the antenna is *radiating in free space*, determine the

(a) radiation resistance of the loop;

(b) loss resistance of the loop (*assume that its value is the same as if the wire were straight*);

(c) input resistance;

(d) input impedance;

(e) radiation efficiency.

a) For $a = \lambda/30$, assume loop is 'small'. Per

(5-24) $R_r = \eta(\pi/6)(k^2a^2)^2 = 376.7303(\pi/6)\left(\left(\frac{2\pi}{\lambda}\right)^2\left(\frac{\lambda}{30}\right)^2\right)^2$

$R_r = 0.379545\,\Omega$

b) Per (2-90b), $R_L = R_{hf} = \frac{l}{P}\sqrt{\frac{\omega\mu_0}{2\sigma}} = \frac{2\pi(\lambda/30)}{2\pi(\lambda/1000)}\sqrt{\frac{2\pi(20\times10^6)4\pi\times10^{-7}}{2(5.7\times10^7)}}$

$R_L = 0.039232\,\Omega$

c) Per (5-33), $Z_{in} = R_{in} + jX_{in} = (R_r + R_L) + j(X_A + X_i)$

$R_{in} = R_r + R_L = 0.379545 + 0.039232 \Rightarrow R_{in} = 0.418777\,\Omega$

d) For Z_{in}, we also need $X_{in} = X_A + X_i$ where,

using (5-37a), $X_A = \omega L_A = \omega\mu_0 a\left[\ln\left(\frac{8a}{b}-2\right]\right.$

$= \omega(4\pi\times10^{-7})\frac{\lambda}{30}\left[\ln\left(\frac{8(\lambda/30)}{\lambda/1000}\right) - 2\right]$

d) cont. $X_A = \omega \left(4\pi \times 10^{-7}\right) \frac{2.9979 \times 10^8}{20 \times 10^6 (30)} \left[\ln\left(\frac{8000}{30}\right) - 2\right]$

$= \omega \left(2.25157 \times 10^{-6}\ H\right)$

$= 2\pi \left(20 \times 10^6\right) \left(2.25157 \times 10^{-6}\right)$

$= \underline{282.94097\ \Omega}$

Per (5-38), $L_i = \frac{a}{\omega b} \sqrt{\frac{\omega \mu_0}{2\sigma}} = \frac{\lambda/30}{2\pi (20 \times 10^6) \frac{\lambda}{1000}} \sqrt{\frac{2\pi (20 \times 10^6) 4\pi \times 10^{-7}}{2 (5.7 \times 10^7)}}$

$= 3.121953 \times 10^{-10}\ H$

$X_i = \omega L_i = 2\pi (20 \times 10^6) 3.121953 \times 10^{-10}$

$= 0.0392316\ \Omega$

$X_{in} = X_A + X_i = 282.94097 + 0.0392316 = \underline{282.9802\ \Omega}$

$\underline{\underline{\bar{Z}_{in} = R_{in} + j X_{in} = 0.4188 + j\, 282.9802\ \Omega}}$

e) Per (2-90), $e_{cd} = \frac{R_r}{R_r + R_L} = \frac{0.379545}{0.418777}$

$\underline{\underline{e_{cd} = 0.90608 \text{ or } 90.608\ \%}}$