5.3 A circular loop, *of loop radius $\lambda/30$ and wire radius $\lambda/1000$*, is used as a transmitting/receiving antenna in a back-pack radio communication system at *10 MHz*. The wire of the loop is made of copper with a conductivity of 5.7×10^7 *S/m*. Assuming the antenna is *radiating in free space*, determine the

(a) radiation resistance of the loop;

(b) loss resistance of the loop (*assume that its value is the same as if the wire were straight*);

(c) input resistance;

(d) input impedance;

(e) radiation efficiency.

a) Radiation resistance since $a = \lambda/30 \ll \lambda$, use (5-24)

$$R_r = \eta\left(\frac{\pi}{6}\right)(kA)^2 = 376.7303\left(\frac{\pi}{6}\right)\left(\frac{2^2\pi^2}{\lambda^2}\frac{\lambda^2}{30^2}\right)^2 \Rightarrow \underline{R_r = 0.379545\ \Omega}$$

b) Loss resistance Per (2-906)

$$R_\ell = R_{hf} = \frac{l}{\rho}\sqrt{\frac{\omega\mu_0}{2\sigma}} = \frac{2\pi\,\lambda/30}{2\pi\,\lambda/1000}\sqrt{\frac{2\pi\,10\times10^6(4\pi\times10^{-7})}{2(5.7\times10^7)}}$$

$$\underline{R_\ell = 0.027741\ \Omega}$$

c) Input resistance see (5-33)

$$R_{in} = R_r + R_\ell = 0.379545 + 0.027741$$

$$\underline{R_{in} = 0.407286\ \Omega}$$

d) Input impedance → Need $X_{in} = X_A + X_i$

see (5-33), (5-37a) & (5-38)

Per (5-37a), $L_A = \mu_0 a\left[\ln\left(\frac{8a}{b}\right) - 2\right] = 4\pi\times10^{-7}\frac{\lambda}{30}\left[\ln\left(\frac{8\frac{\lambda}{30}}{\lambda/1000}\right) - 2\right]$

$$= 4\pi\times10^{-7}\frac{2.9979\times10^8}{(10\times10^6)30}\left[\ln\left(\frac{8000}{30}\right) - 2\right]$$

$$= 4.50315\times10^{-6}\ H$$

See (5-33) → $X_A = \omega L_A = 2\pi(10\times10^6)(4.50315\times10^{-6})$

$$= 282.94097\ \Omega$$

d) cont. Per (5-38), $L_i = \frac{a}{\omega b}\sqrt{\frac{\omega\mu_0}{2\sigma}}$

$$= \frac{\lambda/30}{2\pi(10\times10^6)\frac{\lambda}{1000}}\sqrt{\frac{2\pi(10\times10^6)4\pi\times10^{-7}}{2(5.7\times10^7)}}$$

$$= 4.415108\times10^{-10}\ H$$

$$X_i = \omega L_i = 2\pi(10\times10^6)(4.4151\times10^{-10}) = 0.0277409\ \Omega$$

$$X_{in} = X_A + X_i = 282.94097 + 0.0277409 = 282.96871\ \Omega$$

$$\underline{\bar{Z}_{in} = R_{in} + jX_{in} = 0.407286 + j\,282.968710\ \Omega}$$

e) radiation efficiency

Per (2-90), $e_{cd} = \frac{R_r}{R_r + R_L} = \frac{0.379545}{0.407286}$

$$\underline{e_{cd} = 0.931888 \text{ or } 93.1888\ \%}$$