A thin lossless vertical 0.25λ monopole at the origin in free space is fed through an infinite ground (perfect electrical conductor) plane at $z = 0$ with a feed current $I_0 = 1\angle 0°$ A. Determine the: (a) vector far-field phasor electric field, (b) vector far-field phasor magnetic field, (c) vector radiated time-average power density (Poynting vector), (d) radiation intensity, (e) power radiated, (f) maximum directivity (unitless & dBi), and (g) radiation resistance. [Hint: What would be the length $\ell$ of the image equivalent dipole?]

Free space $\Rightarrow \eta = \eta_0 = 376.7303\ \Omega$

a) Per notes, the fields for a monopole over a PEC ground plane are identical to the image equivalent dipole in the $z>0$ halfspace. For a $\lambda/2$ dipole

$$(4\text{-}84)\quad E_\theta = j\eta \frac{I_0 e^{-jkr}}{2\pi r}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]$$

$$= j\,376.73\,\frac{1\angle 0°}{2\pi}\,\frac{e^{-jkr}}{r}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]$$

$$\bar{E}_{FF} = \hat{a}_\theta\, j\, 59.9585\,\frac{e^{-jkr}}{r}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]\ \text{V/m}$$

$$r>0,\ 0\le\phi<2\pi,\ 0\le\theta<\pi/2$$

b) $$(4\text{-}85)\quad H_\phi = j\frac{I_0 e^{-jkr}}{2\pi r}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]$$

$$= j\,\frac{1\angle 0°}{2\pi}\,\frac{e^{-jkr}}{r}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]$$

$$\bar{H}_{FF} = \hat{a}_\phi\, j\, 0.159155\,\frac{e^{-jkr}}{r}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]\ (\text{A/m})$$

$$r>0,\ 0\le\phi<2\pi,\ 0\le\theta<\pi/2$$

c) Per (2-8), $\bar{W}_{ave} = \frac{1}{2} Re\{\bar{E} \times \bar{H}^*\}$

$$\bar{W}_{ave} = \frac{1}{2} Re\left\{ \hat{a}_\theta \, j\, 59.96 \frac{e^{-jkr}}{r}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right] \times \hat{a}_\phi (-j)\, 0.159 \frac{e^{+jkr}}{r}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]\right\}$$

$$= \frac{1}{2} Re\left\{ \hat{a}_r (1)\, 9.54269 \frac{e^0}{r^2}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]^2\right\}$$

$$\underline{\underline{\bar{W}_{ave} = \hat{a}_r \frac{4.771345}{r^2}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]^2 \left(W/m^2\right)}}$$

$$\underline{\underline{r > 0,\ 0 \le \phi < 2\pi,\ 0 \le \theta < \pi/2}}$$

d) Per (2-12), $U = r^2 W_{rad}$

$$\underline{\underline{U(\theta) = 4.771345\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]^2 \left(W/sr\right)}}$$

$$\underline{\underline{r > 0,\ 0 \le \phi < 2\pi,\ 0 \le \theta < \pi/2}}$$

e) (2-13) $P_{rad} = \oiint U \, d\Omega$

$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} 4.77\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]^2 \sin\theta \, d\theta\, d\phi$$

$$P_{rad} = 4.771345\,(2\pi - 0)\underbrace{\int_{\theta=0}^{\pi/2} \frac{\cos^2(\frac{\pi}{2}\cos\theta)}{\sin\theta}\, d\theta}_{\to 0.6094133 \text{ MathCad}}$$

$$\underline{\underline{P_{rad} = 18.26975 \text{ W}}}$$

f) (2-16a) $D_{max} = \frac{4\pi U_{max}}{P_{rad}}$

$U_{max} = U(\pi/2) = 4.771345$

$D_{max} = \frac{4\pi(4.771345)}{18.26975}$

$\underline{\underline{D_{max} = 3.281845 = 5.1612\ dBi}}$

(Note: $2.15 + 3.01 = 5.16\ dBi$)

$2.15$ ← Dipole; $3.01$ ← double, $10\log 2$

g) (2-76) $P_r = \frac{1}{2}|I_g|^2 R_r$

$R_r = \frac{2P_{rad}}{|I_s|^2} = \frac{2(18.26975)}{|1\angle 0|^2}$

$\underline{\underline{R_r = 36.5395\ \Omega}}$

(Note: $\frac{1}{2}(73) = 36.5\ \Omega$ ← $\frac{1}{2}$ Dipole $R_r$)