4.46 Repeat Problem 4.45 for a center-fed $5\lambda/8$ dipole.

4.45 A lossless, resonant, center-fed $3\lambda/4$ linear dipole, radiating in free-space is attached to a balanced, lossless transmission line whose characteristic impedance is 300 ohms. Assuming $a = 0.03\lambda$, calculate the:

(a) radiation resistance (referred to the current maximum)

(b) input impedance (referred to the input terminals)

(c) VSWR on the transmission line

For parts (a) and (b) use the computer program **Dipole** at the end of the chapter.

- Let $\ell/a = 64$. Do NOT ignore the reactance in this case. [Hint: See 4-70a & 8-60b.] For part (a) Also, find radiation reactance. Check/compare with results using NEC-2 assuming $f = 299.8$ MHz and with $\Delta/a \sim 4$. Note: You may use MathCad, Matlab, … instead of computer program at end of chapter for analytic results.

a) Using (4-67) + (4-70)

$$R_r = \frac{2P_{rad}}{|I_0|^2} = \frac{\eta}{2\pi}\int_{\theta=0}^{\pi}\frac{\left[\cos\left(\frac{k\ell}{2}\cos\theta\right)-\cos\left(\frac{k\ell}{2}\right)\right]^2}{\sin\theta}\,d\theta$$

where $k\ell = \left(\frac{2\pi}{\lambda}\right)\frac{5\lambda}{8} = 1.25\pi$ and $\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.73031\,\Omega$

$$R_r = \frac{376.73}{2\pi}\int_{\theta=0}^{\pi}\frac{\left[\cos(0.625\pi\cos\theta)-\cos(0.625\pi)\right]^2}{\sin\theta}\,d\theta \quad \leftarrow \text{MathCad}$$

$$= 59.481(2.1990257) = \underline{\underline{131.85027\,\Omega}}$$

Per (4-70a) or (8-60b)

$$X_r = X_m = \frac{\eta}{4\pi}\Big\{2S_i(1.25\pi)+\cos(1.25\pi)\left[2S_i(1.25\pi)-S_i(2.5\pi)\right] - \sin(1.25\pi)\left[2C_i(1.25\pi)-C_i(2.5\pi)-C_i\left(\frac{2.5\pi}{64^2}\right)\right]\Big\}$$

$$\underline{\underline{X_r = 176.423\,\Omega}}$$

b) To get input resistance + reactance, use (4-79)/(8-61a) + (8-61b)

$$R_{in} = \frac{R_r}{\sin^2(k\ell/2)} = \frac{131.85}{\sin^2(0.625\pi)} = \underline{154.4722\,\Omega} + X_{in} = \frac{176.423}{\sin^2(0.625\pi)} = \underline{\underline{206.69\,\Omega}}$$

$$\hookrightarrow Z_{in} = R_{in} + jX_{in} = \underline{\underline{154.47 + j206.69\,\Omega}}$$

c) $$\Gamma = \frac{Z_{in}-Z_0}{Z_{in}+Z_0} = \frac{(154.47+j206.69)-300}{(154.47+j206.69)+300} = 0.5063\angle 100.69^\circ$$

$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.5063}{1-0.5063} = \underline{\underline{3.051}}$$

Check calculations using MATLAB-

```
% EE 483/583 problem 4.46(p4_46.m)
% Calculate Rr & Xr for (5/8)lambda linear dipole at current maxima
% Then calculate Zin for antenna at feed.  Assume l/a = 64.
clear;clc;close all;
kl = (5/8)*2*pi;    % wavenumber*l product
la = 64; % antenna length to radius ratio
eta = 376.730313461; % intrinsic impedance of free space
C = 0.5772156649015328606065120900824024310421; % Euler's constant
Z0 = 300; % Characteristic impedance of transmission line
% Use (4-70)/(8-60a) to get radiation resistance at current maxima
Rr = (eta/(2*pi))*(C+log(kl)-cosint(kl)+0.5*sin(kl)*(sinint(2*kl)-2*sinint(kl))...
    +0.5*cos(kl)*(C + log(kl/2) + cosint(2*kl) - 2*cosint(kl)))
% Use (4-70a)/(8-60b) to get reactance at current maxima
Xr = (eta/(4*pi))*(2*sinint(kl)+cos(kl)*(2*sinint(kl)-sinint(2*kl))...
    -sin(kl)*(2*cosint(kl)-cosint(2*kl)-cosint(2*kl/la^2)))
% Use (4-79)/(8-61a) to get input resistance at feed
Rin = Rr/(sin(0.5*kl)*sin(0.5*kl))
% Use (8-61b) to get reactance at feed
Xin = Xr/(sin(0.5*kl)*sin(0.5*kl))
Zin = Rin + j*Xin
% Calculate input reflection coefficient & VSWR
Gamma = (Zin-Z0)/(Zin+Z0)
Gmag = abs(Gamma)
VSWR = (1+Gmag)/(1-Gmag)
```

$\boldsymbol{R_r = 131.8503\ \Omega}$ **and** $\boldsymbol{X_r = 176.4231\ \Omega}$

$\boldsymbol{R_{\text{in}} = 154.4722\ \Omega}$ **and** $\boldsymbol{X_{\text{in}} = 206.6925\ \Omega}$

$\Rightarrow$ $\boxed{\boldsymbol{Z_{\text{in}} = 154.47 + j206.6925\ \Omega}}$

$\boldsymbol{\Gamma}$ **= Gamma = -0.0939 + 0.4375i , Gmag = 0.5063, Gangle = 100.6927**

$\Rightarrow$ $\boxed{\textbf{VSWR = 3.0511}}$

Compare/check against NEC-2 simulation-

Wavelength $\lambda = c/f = 2.998*10^8/299.8*10^6$ $\Rightarrow$ **$\lambda = 1$ m**

Antenna length $\ell = (5/8)\lambda = (5/8)1$ $\Rightarrow$ **$\ell = 0.625$ m**

Antenna radius $a = \ell/64 = 0.625/64$ $\Rightarrow$ **$a = 0.009765625$ m**

Segment length Δ selection

- Is wire 'thin', i.e., $2\pi a/\lambda \ll 1$? $\Rightarrow$ Yes, $2\pi(0.009765625)/1 = 0.061359 \ll 1$ $\therefore$
- Want $\Delta < \lambda/10$ and $\Delta > \lambda/1000$, $\Rightarrow 0.001 \text{ m} < \Delta < 0.1 \text{ m}$
- Want minimum $\Delta/a > 2 \Rightarrow \Delta > 2a = 0.01953125$ m which implies 0.625/0.039 ~ 32 segments.
- **But** $\Delta/a > 8$ is better $\Rightarrow \Delta > 8a = 0.078125$ m which implies 0.625/0.078125 ~ 8 segments.
- Another consideration is that we want a center-fed dipole $\Rightarrow$ odd integer number of segments.
- choose **15** segments, $\Delta = \ell/15 = 0.625/15 \Rightarrow$ **$\Delta = 0.041667$ m** $\Rightarrow \Delta/a = 4.3$. **Use EK command.**

NEC input file-

```
CM p4_46_la64.txt
CM Lossless (5/8)lambda center-fed dipole antenna
CM at f = 299.8 MHz (lambda = 1 m) with L/a = 64
CM i.e., length = L = 5/8(1) = 0.625 m, and
CM radius a = 0.625/64 = 0.009765625 m.
CM Used 15 segments.  DRIVEN SEGMENT IS #8.
CE
GW 1  15 0.0 0.0 -0.3125 0.0 0.0 0.3125 0.009765625
GE 0  ! No ground plane
EK 0  ! Use extended kernel in simulation
FR 0 1 0 0 299.8 0.0
EX 0 1 8 00 1.0 0.0  ! Center-fed
XQ 0   ! Start simulation
EN
```

Excerpts from NEC output file-

<snip>

```
                              FREQUENCY= 2.9980E+02 MHZ
                              WAVELENGTH= 1.0000E+00 METERS
```

<snip>

```
                         - - - ANTENNA INPUT PARAMETERS - - -
TAG SEG  VOLTAGE (VOLTS)   CURRENT (AMPS)          IMPEDANCE (OHMS)       POWER
NO. NO. REAL    IMAG.      REAL         IMAG.      REAL        IMAG.      (WATTS)
1   8 1.00E+00 0.0E+00 2.86152E-03-1.75308E-03 2.54096E+02 1.55669E+02
2.86152E-03-1.75308E-03 1.43076E-03

                         - - - CURRENTS AND LOCATION - - -
                            DISTANCES IN WAVELENGTHS

SEG TAG COORD OF SEG CENTER SEG.     - - - CURRENT (AMPS) - - -
NO. NO.  X   Y      Z       LENGTH    REAL       IMAG.       MAG.       PHASE
1   1   0.0 0.0 -0.2917  0.04167  4.8037E-04 -1.0929E-03 1.1938E-03  -66.273
2   1   0.0 0.0 -0.2500  0.04167  1.0464E-03 -2.1994E-03 2.4356E-03  -64.558
3   1   0.0 0.0 -0.2083  0.04167  1.5426E-03 -2.9715E-03 3.3481E-03  -62.565
4   1   0.0 0.0 -0.1667  0.04167  1.9842E-03 -3.4624E-03 3.9907E-03  -60.185
5   1   0.0 0.0 -0.1250  0.04167  2.3526E-03 -3.6429E-03 4.3365E-03  -57.146
6   1   0.0 0.0 -0.0833  0.04167  2.6302E-03 -3.4821E-03 4.3638E-03  -52.934
7   1   0.0 0.0 -0.0417  0.04167  2.8029E-03 -2.8615E-03 4.0056E-03  -45.593
8   1   0.0 0.0  0.0     0.04167  2.8615E-03 -1.7531E-03 3.3558E-03  -31.493
9   1   0.0 0.0  0.0417  0.04167  2.8029E-03 -2.8615E-03 4.0056E-03  -45.593
10  1   0.0 0.0  0.0833  0.04167  2.6302E-03 -3.4821E-03 4.3638E-03  -52.934
11  1   0.0 0.0  0.1250  0.04167  2.3526E-03 -3.6429E-03 4.3365E-03  -57.146
12  1   0.0 0.0  0.1667  0.04167  1.9842E-03 -3.4624E-03 3.9907E-03  -60.185
13  1   0.0 0.0  0.2083  0.04167  1.5426E-03 -2.9715E-03 3.3481E-03  -62.565
14  1   0.0 0.0  0.2500  0.04167  1.0464E-03 -2.1994E-03 2.4356E-03  -64.558
15  1   0.0 0.0  0.2917  0.04167  4.8037E-04 -1.0929E-03 1.1938E-03  -66.273

- - - POWER BUDGET - - -
INPUT POWER   = 1.4308E-03 WATTS
RADIATED POWER= 1.4308E-03 WATTS
STRUCTURE LOSS= 0.0000E+00 WATTS
NETWORK LOSS  = 0.0000E+00 WATTS
EFFICIENCY    = 100.00 PERCENT
```

<snip>

- From NEC, $\boldsymbol{Z_{in}}$ **= 254.096 + *j* 155.669 Ω** (not too close to analytic **154.47 + *j* 206.69 Ω**) at the feed point (seg #8) with $\overline{I}_{in} = 3.35583\angle -31.493°$ mA. However, the maximum currents are $\overline{I}_{max} = 4.3638\angle -52.934°$ mA occurring at segments 6 & 10, $P_{in} = P_{rad}$ = 1.4308 mW, P_{loss} = 0, & efficiency = 100 %.

$$R_r = \frac{P_{rad}}{0.5\,|\overline{I}_{max}|^2} = \frac{0.0014308}{0.5(0.0043638)^2} \quad \Rightarrow \quad \underline{R_r = 150.27\ \Omega}$$ (near analytic R_r = 131.85 Ω),

$$\text{Check-} \quad R_{in} = \frac{P_{rad}}{0.5\,|\overline{I}_{in}|^2} = \frac{0.0014308}{0.5(0.00335583)^2} \quad \Rightarrow \quad \underline{R_{in} = 254.10\ \Omega}$$

<u>Comments</u>-

- A close reading of the text reveals that the analytic formulas assume very thin wires, whereas ℓ/a = 64 leads to $2\pi a/\lambda$ = 0.061359 which is borderline.
- Reality- input impedance of dipoles, particularly reactance, is very sensitive to feed structure.
- In NEC, we are using Δ = 0.041667 m and a = 0.009765 m, or Δ/a = 4.3 which requires the extended kernel EK command. Changing the number and length of segments can have a significant effect on results.