4.45 A lossless, resonant, center-fed $3\lambda/4$ linear dipole, radiating in free-space is attached to a balanced, lossless transmission line whose characteristic impedance is 300 ohms. Assuming $a = 0.03\lambda$, calculate the:

(a) radiation resistance (referred to the current maximum)

(b) input impedance (referred to the input terminals)

(c) VSWR on the transmission line

For parts (a) and (b) use the computer program **Dipole** at the end of the chapter.

- Antenna NOT resonant (i.e., we'll consider reactance). Hence, a) Also find reactance. c) Also find reflection coefficient (phasor format w/ angle in degrees). Note: You may use MathCad, Matlab, … instead of computer program **Dipole** at end of chapter.

a) Calculate R_r using equation (4-70) or by doing numerical integral given in notes-

(4-70) $R_r = (\eta/2\pi)\{C + \ln(kl) - C_i(kl) + 0.5\sin(kl)[S_i(2kl) - 2S_i(kl)] + 0.5\cos(kl)[C + \ln(kl/2) + C_i(2kl) - 2C_i(kl)]\}$

or, (notes) $$R_r = \frac{\eta}{2\pi}\int_0^{\pi}\frac{[\cos(0.5kl\cos(\theta)) - \cos(0.5kl)]^2}{\sin(\theta)}d\theta$$

(4-70a) $X_m = (\eta/4\pi)\{2S_i(kl) + \cos(kl)[2S_i(kl) - S_i(2kl)] - \sin(kl)[2C_i(kl) - C_i(2kl) - C_i(2ka^2/l)]\}$

where $\eta = 376.7303\ \Omega$, Euler's Constant $C = 0.5772156649$, and $kl = \frac{2\pi}{\lambda}(0.75\lambda) = 1.5\pi$.

Using Matlab (see code below), this yielded-

Rr1 = 1.85680060809273e+02 = Rr = 1.85680060809273e+02 ⇒ **R_r = 185.68006 Ω**

Xm =1.926634471383175e+02 ⇒ **X_m = 192.66345 Ω**

b) Calculate input impedance using equation (4-79) to reference R_r to input (same for X_m), i.e., $R_{in} = R_r/\sin^2(0.5kl)$ and $X_{in} = X_m/\sin^2(0.5kl)$ where $0.5kl = 0.75\pi$. Using Matlab (see code below), this yielded-

Rin = 3.713601216185460e+02, Xin = 3.853268942766349e+02, &

Zin = 3.713601216185460e+02 + 3.853268942766349e+02i

⇒ **Z_{in} = 371.36012 + j385.32689 Ω**

c) To get the VSWR, first calculate the reflection coefficient (confirmed w/ Matlab)

$\Gamma = (Z_{in} - Z_0)/(Z_{in} + Z_0) = [(371.4+j385.3)-300]/[(371.4+j385.3)+300]$

⇒ **$\Gamma = 0.506\angle 49.65°$**

and then the find the standing wave ratio (confirmed w/ Matlab)

$\text{VSWR} = (1 + |\Gamma|)/(1 - |\Gamma|) = (1 + 0.506)/(1 - 0.506)$ ⇒ **VSWR = 3.05064**.

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% EE 483/583 problem 4.45(p4_45.m)
% For a lossless 0.75 lambda linear dipole,
% calculate Rr, Xm, Zin, Gamma, & VSWR
% where radius a = 0.03 lambda
%
clear;clc;close all;
format long
kl = 2*pi*0.75;      % wavenumber*l product
al = 0.03/0.75;      % ratio of antenna radius to length
eta = 376.730313461; % intrinsic impedance of free space
Z0 = 300;            % characteristic impedance of transmission line
C = 0.57721566490153286060651209008240243104215; % Euler's constant
% Calculate Rr using numerical integration
fun = @(x) (cos(0.5*kl*cos(x))-cos(0.5*kl)).^2./sin(x);
Rr1 = (eta/(2*pi))*integral(fun,0, pi)
% Calculate Rr using (4-70) of text [Balanis Ant. Theory 4th edn]
Rr = (eta/(2*pi))*(C+log(kl)-cosint(kl)+0.5*sin(kl)*(sinint(2*kl)-2*sinint(kl))...
    +0.5*cos(kl)*(C + log(kl/2) + cosint(2*kl) - 2*cosint(kl)))
% Calculate Xm using (4-70a) of text [Balanis Ant. Theory 4th edn]
Xm = (eta/(4*pi))*(2*sinint(kl)+cos(kl)*(2*sinint(kl)-sinint(2*kl))...
    -sin(kl)*(2*cosint(kl)-cosint(2*kl)-cosint(2*kl*al^2)))
Rin = Rr/sin(0.5*kl)^2;            % Translate Rr to input
Xin = Xm/sin(0.5*kl)^2;            % Translate Xm to input
Zin = Rin + j*Xin                  % Input impedance
Gamma = (Zin-Z0)/(Zin+Z0) % Reflection coeff.
Gamma_mag = abs(Gamma) % Magnitude of reflection coeff.
VSWR = (1+Gamma_mag)/(1-Gamma_mag) % Calculate VSWR
```