4.29 The *approximate far zone electric field* radiated by a *very thin wire* linear dipole of length l, positioned symmetrically along the z-axis, is given by

$$E_\theta \simeq C_o \sin^{1.5}\theta \, \frac{e^{-jkr}}{r}$$

where C_o is a constant. Determine the:

(a) Exact directivity (*dimensionless* and *in dB*).

(b) Approximate directivity (*dimensionless* and *in dB*) using an approximate but appropriate formula (*state the formula you are using*).

(c) Length of the dipole (*in wavelengths*).

(d) Input impedance of the dipole. Assume the wire radius a is very small ($a \ll \lambda$).

a) From notes, $\overline{W}_{ave} = \hat{a}_K \frac{|\overline{E}|^2}{2\eta}$ where $\hat{a}_K = \hat{a}_r$

$$\overline{W}_{ave} = \hat{a}_r \frac{1}{2\eta} C_o \sin^{1.5}\theta \, \frac{e^{-jkr}}{r} \left(C_o \sin^{1.5}\theta \, \frac{e^{+jkr}}{r} \right)$$

$$= \hat{a}_r \frac{C_o^2 \sin^3\theta}{2\eta r^2} = \overline{W}_{rad}$$

Per (2-12), $U = r^2 W_{rad} = \dfrac{C_o^2 \sin^3\theta}{2\eta}$

Per (2-13), $P_{rad} = \oiint U \, d\Omega = \dfrac{C_o^2}{2\eta} \displaystyle\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \sin^3\theta \sin\theta \, d\theta \, d\phi$

$$P_{rad} = \frac{C_o^2}{2\eta} \int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi} \sin^4\theta \, d\theta$$

$$= \frac{C_o^2}{2\eta} \, \phi \Big|_{\phi=0}^{2\pi} \left[\frac{3\theta}{8} - \frac{\sin 2\theta}{4} + \frac{\sin 4\theta}{32} \right] \Bigg|_{\theta=0}^{\pi}$$

$$= \frac{C_o^2}{2\eta} (2\pi - 0) \left[\left(\frac{3\pi}{8} - 0 + 0 \right) - (0 - 0 + 0) \right]$$

$$= \frac{3\pi^2 C_o^2}{8\eta}$$

Per (2-16), $D = \dfrac{4\pi U}{P_{rad}} = \dfrac{4\pi \dfrac{C_o^2 \sin^3\theta}{2\eta}}{3\pi^2 C_o^2 / 8\eta} = \dfrac{16 \sin^3\theta}{3\pi}$

a) $D(\theta) = 1.69765 \sin^3\theta \qquad 0 \le \theta \le 180°$

@ $\theta_{max} = 90°$, $D_0 = D_{max} = 1.69765 = 2.2985$ dBi

b) Using $U = \frac{C_0^2 \sin^3\theta}{2\eta} \Rightarrow U_{norm} = \sin^3\theta$ w/ $\theta_{max} = 90°$

Find θ_h. $U_{norm}(\theta_h) = 0.5 = \sin^3\theta_h$

$\Rightarrow \theta_h = \sin^{-1} 0.5^{1/3} = 52.5327°$ or $127.4673°$

$HPBW = 127.4673° - 52.5327° = 74.9346°$

w/ no ϕ dependence, this is an omnidirectional antenna. $\Rightarrow$ Can use (2-33a) McDonald or (2-33b) Pozar.

(2-33a) $D_0 \simeq \frac{101}{HPBW(\deg) - 0.0027\, HPBW(\deg)^2}$

$\simeq \frac{101}{74.9346 - 0.0027(74.9346)^2}$

$D_0 \simeq 1.6897 = 2.278$ dBi

or

(2-33b) $D_0 \simeq -172.4 + 191\sqrt{0.818 + 1/HPBW(\deg)}$

$\simeq -172.4 + 191\sqrt{0.818 + 1/74.9346}$

$D_0 \simeq 1.7502 = 2.431$ dBi

c) Per section 4.6 eqn (4-87), $\ell = \lambda/2$

d) Per (4-93a), $Z_{in} = 73 + j42.5\ \Omega$