4.18 For an antenna with a maximum linear dimension of D, find the inner and outer boundaries of the Fresnel region so that the maximum phase error does not exceed **(f)** $\pi/\mathbf{20}$ rad

f) phase error of $\pi/20 = 9^\circ$

<u>Outer boundary (Fresnel – Far-zone/Fraunhofer)</u>

Per (4-41), $R = r - z'\cos\theta + \frac{1}{r}\left(\frac{z'^2}{2}\sin^2\theta\right) + \frac{1}{r^2}\left(\frac{z'^3}{2}\cos\theta\sin^2\theta\right) + \ldots$

* Here, we will keep the first two terms, $R \simeq r - z'\cos\theta$.
* Therefore, the '$\frac{1}{r}\left(\frac{z'^2}{2}\sin^2\theta\right)$' term is the largest error term.

* For the worst case or maximum error, select $\theta = \pi/2 = 90^\circ$ and $z' = D/2$.

* Then, the maximum phase error in the e^{-jkR} term is set equal to $\pi/20 \Rightarrow k\frac{1}{r}\left(\frac{(D/2)^2}{2}\sin^2 90^\circ\right) = \pi/20$.

* Solving for $r = r_{outer}$, we get

$$r_{outer} = \frac{20}{\pi} k \frac{D^2}{4(2)} 1^2 = \frac{20}{\pi}\left(\frac{2\pi}{\lambda}\right)\frac{D^2}{8}$$

$$\underline{\underline{r_{outer} = 5\frac{D^2}{\lambda}}}$$

Inner boundary (Reactive near-field – Fresnel)

* Here, we will keep the first three terms of (4-41),

$$R \simeq r - z'\cos\theta + \frac{1}{r}\left(\frac{z'^2}{2}\sin^2\theta\right).$$

* Therefore, the largest error term is now

'$\frac{1}{r^2}\left(\frac{z'^3}{2}\cos\theta\sin^2\theta\right)$'.

* For the worst case or maximum error, select $\theta = \tan^{-1}\sqrt{2} = 54.7356°$ and $z' = D/2$.

* Then, the maximum phase error in the e^{-jkR} term is set equal to $\pi/20$:

$$k\,\frac{1}{r^2}\left(\frac{(D/2)^3}{2}\cos(54.7356°)\sin^2(54.7356°)\right) = \pi/20$$

Solving for $r = r_{inner}$, we get

$$\frac{2\pi}{\lambda}\,\frac{1}{r_{inner}^2}\,\frac{D^3}{8(2)}\,\frac{1}{\sqrt{3}}\left(\frac{2}{3}\right) = \pi/20$$

$$r_{inner} = \sqrt{\frac{20}{\pi}\left(\frac{2\pi}{\lambda}\right)\frac{D^3}{24\sqrt{3}}}$$

$$\underline{\underline{r_{inner} = 0.98094\sqrt{D^3/\lambda}}}$$

Fresnel region $\boxed{0.9809\sqrt{D^3/\lambda} < r < 5\frac{D^2}{\lambda}}$ w/ max phase error of $\pi/20$