4.18 For an antenna with a maximum linear dimension of D, find the inner and outer boundaries of the Fresnel region so that the maximum phase error does not exceed (a) $\pi/18$ rad

e) phase error of $\pi/18 = 10^\circ$

Outer boundary (Fresnel – Far-zone/Fraunhofer)

Per (4-41), $R = r - z'\cos\theta + \frac{1}{r}\left(\frac{z'^2}{2}\sin^2\theta\right) + \frac{1}{r^2}\left(\frac{z'^3}{2}\cos\theta\sin^2\theta\right) + \cdots$

* Here, we will keep the first two terms, $R \simeq r - z'\cos\theta$.

* Therefore, the '$\frac{1}{r}\left(\frac{z'^2}{2}\sin^2\theta\right)$' term is the largest error term.

* For the worst case or maximum error, select $\theta = \pi/2 = 90^\circ$ and $z' = \ell/2$.

* Then, the maximum phase error in the e^{-jkR} term is set equal to $\pi/18 \Rightarrow k\frac{1}{r}\left(\frac{(\ell/2)^2}{2}\sin^2 90^\circ\right) = \pi/18$.

* Solving for $r = r_{outer}$, we get

$$r_{outer} = \frac{18}{\pi} k \frac{\ell^2}{4(2)} 1^2 = \frac{18}{\pi}\left(\frac{2\pi}{\lambda}\right)\frac{\ell^2}{8}$$

$$r_{outer} = \frac{9\ell^2}{2\lambda} = 4.5\frac{\ell^2}{\lambda}$$

Inner boundary (Reactive near-field – Fresnel)

* Here, we will keep the first three terms of (4-41),

$$R \simeq r - z'\cos\theta + \frac{1}{r}\left(\frac{z'^2}{2}\sin^2\theta\right).$$

* Therefore, the largest error term is now

$$\frac{1}{r^2}\left(\frac{z'^3}{2}\cos\theta\sin^2\theta\right)'.$$

* For the worst case or maximum error,

select $\theta = \tan^{-1}\sqrt{2} = 54.7356^\circ$ and $z' = D/2$.

* Then, the maximum phase error in the e^{-jkR} term is set equal to $\pi/8$:

$$k\,\frac{1}{r^2}\left(\frac{(D/2)^3}{2}\cos(54.7356^\circ)\sin^2(54.7356^\circ)\right) = \pi/8$$

Solving for $r = r_{inner}$, we get

$$\frac{2\pi}{\lambda}\,\frac{1}{r_{inner}^2}\,\frac{D^3}{8(2)}\,\frac{1}{\sqrt{3}}\left(\frac{2}{3}\right) = \pi/8$$

$$r_{inner} \simeq \sqrt{\frac{18}{\pi}\left(\frac{2\pi}{\lambda}\right)\frac{D^3}{24\sqrt{3}}}$$

$$r_{inner} = 0.9306\sqrt{D^3/\lambda}$$

Fresnel region

$$\boxed{0.9306\sqrt{\frac{D^3}{\lambda}} < r < 4.5\,\frac{D^2}{\lambda}}$$

w/ max phase error of $\pi/8$