4.18 For an antenna with a maximum linear dimension of *D*, find the inner and outer boundaries of the Fresnel region so that the maximum phase error does not exceed (c) 18°

c) $18° = 18°\left(\pi/180°\right) = \pi/10$ ← max. error of kr term

<u>Outer boundary</u> (i.e., Fresnel to far-zone/Fraunhofer)

Per (4-41), $R = r - z'\cos\theta + \frac{1}{r}\left(\frac{z'^2}{2}\sin^2\theta\right) + \frac{1}{r^2}\left(\frac{z'^3}{2}\cos\theta\sin^2\theta\right) \ldots$

If we keep the first two terms ($R = r - z'\cos\theta$), then, the $\frac{1}{r}\left(\frac{z'^2}{2}\sin^2\theta\right)$ term will be the largest error.

Note, this term is largest when $\theta = \pi/2 = 90°$.

Equate with maximum desired error of $\pi/10$

$$k\frac{1}{r}\left(\frac{z'^2}{2}\underbrace{\sin^2\pi/2}_{1}\right) = \pi/10$$

Now, along our dipole, this term is worst when $z' = l/2 = D/2$ which yields

$$k\frac{1}{r_{outer}}\frac{D^2/4}{2} = \pi/10$$

$$r_{outer} \geq \frac{10}{\pi}k\frac{D^2}{8} = \frac{10}{\pi}\left(\frac{2\pi}{\lambda}\right)\frac{D^2}{8} = \frac{20}{8}\frac{D^2}{\lambda}$$

$$\underline{\underline{r_{outer} \geq 2.5\,D^2/\lambda}}$$

c) cont. Inner Boundary (i.e., Near-field to Fresnel)

Now, we will keep the first three terms of R from (4-41), i.e., $R \simeq r - z'\cos\theta + \frac{1}{r}\left(\frac{z'^2}{2}\sin^2\theta\right)$. This makes $\frac{1}{r^2}\left(\frac{z'^3}{2}\cos\theta\sin^2\theta\right)$ the largest error term.

This term is largest wrt θ when

$$\theta = \tan^{-1}(\pm\sqrt{2}) = 54.7^\circ \quad (4\text{-}50a)$$

and $z' = \ell/2 = D/2$

Now, setting the phase error equal to $\pi/10$, yields

$$k\frac{1}{r_{inner}^2}\left(\frac{D^3/2^3}{2}\cos 54.7^\circ \sin^2 54.7^\circ\right) = \pi/10$$

$$k\frac{1}{r_{inner}^2}\left(\frac{D^3}{16}\frac{1}{\sqrt{3}}\,\frac{2}{3}\right) = \pi/10$$

$$r_{inner}^2 = \frac{10}{\pi}\,k\,\frac{2}{3\sqrt{3}}\,D^3/16$$

$$= \frac{10}{\pi}\,\frac{2\pi}{\lambda}\,\frac{1}{24\sqrt{3}}\,D^3$$

$$= \frac{20}{24\sqrt{3}}\,D^3/\lambda$$

$$\underline{\underline{r_{inner} \simeq \sqrt{\frac{5}{6\sqrt{3}}\,D^3/\lambda} = 0.69363\sqrt{D^3/\lambda}}}$$