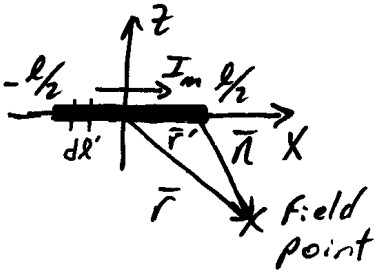
4.10 For the infinitesimal magnetic dipole of Problem 4.9, find the far-zone fields when the element is placed along the (a) x-axis.

4.9 An infinitesimal magnetic dipole of constant current I_m and length l is symmetrically placed about the origin along the z-axis. Find the
(a) spherical **E**- and **H**-field components radiated by the dipole in all space
(b) directivity of the antenna



Per (3-54), $\bar{F} = \frac{\epsilon}{4\pi}\int_c \bar{I}_m \frac{e^{-jkR}}{R}\,dl'$

$\bar{I}_m = \hat{a}_x I_m \quad -l/2 \le x' \le l/2$

$dl' = dx'$

For $l \ll \lambda$, let $\bar{R} = \bar{r} - \bar{r}' \simeq \bar{r} \Rightarrow R \simeq r$

$$\bar{F} = \frac{\epsilon}{4\pi}\int_{x'=-l/2}^{l/2} \hat{a}_x I_m \frac{e^{-jkr}}{r}\,dx' = \hat{a}_x \frac{\epsilon I_m l}{4\pi}\frac{e^{-jkr}}{r}$$

$$\bar{F} = (\sin\theta\cos\phi\,\hat{a}_r + \cos\theta\cos\phi\,\hat{a}_\theta - \sin\phi\,\hat{a}_\phi)\frac{\epsilon I_m l}{4\pi}\frac{e^{-jkr}}{r}$$

Far-zone fields

Per (3-59a), $H_r \simeq 0$, $H_\theta = -j\omega F_\theta$, & $H_\phi = -j\omega F_\phi$

$$\bar{H}_{FF} = \frac{-j\omega\epsilon I_m l}{4\pi}\frac{e^{-jkr}}{r}(\cos\theta\cos\phi\,\hat{a}_\theta - \sin\phi\,\hat{a}_\phi)$$

Per (3-59b), $E_r \simeq 0$, $E_\theta \simeq -j\omega\eta F_\phi$, & $E_\phi = +j\omega\eta F_\theta$

$$\bar{E}_{FF} = \frac{j\omega\eta\epsilon I_m l}{4\pi}\frac{e^{-jkr}}{r}(\sin\phi\,\hat{a}_\theta + \cos\theta\cos\phi\,\hat{a}_\phi)$$