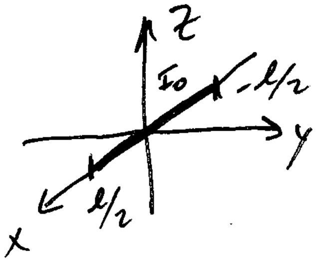
4.3 Repeat Problem 4.1 using the procedure of Example 4.5.

4.1 A horizontal infinitesimal electric dipole of constant current I_0 is placed symmetrically about the origin and directed along the x-axis. Derive the

(a) far-zone fields radiated by the dipole

(b) directivity of the antenna



For an infinitesimal dipole along the z-axis, we got (4-4) $\bar{A} = \hat{a}_z \frac{\mu I_0 l}{4\pi r} e^{-jkr}$. Per example 4.5 of the text, for the same dipole along the x-axis, we get $\bar{A} = \hat{a}_x \frac{\mu I_0 l}{4\pi r} e^{-jkr}$

Now, $\hat{a}_x = \sin\theta\cos\phi\,\hat{a}_r + \cos\theta\cos\phi\,\hat{a}_\theta - \sin\phi\,\hat{a}_\phi$, which gives

$$\bar{A} = \frac{\mu I_0 l}{4\pi r} e^{-jkr}\left(\sin\theta\cos\phi\,\hat{a}_r + \cos\theta\cos\phi\,\hat{a}_\theta - \sin\phi\,\hat{a}_\phi\right)$$

a) In the far-zone, per (3-58a), $\bar{E}_{FF} \simeq -j\omega A_\theta\,\hat{a}_\theta - j\omega A_\phi\,\hat{a}_\phi$

and, per (3-58b), $\bar{H}_{FF} = \frac{j\omega}{\eta} A_\phi\,\hat{a}_\theta - \frac{j\omega}{\eta} A_\theta\,\hat{a}_\phi$

$$\boxed{\begin{aligned}\bar{E}_{FF} &= \frac{-j\omega\mu I_0 l}{4\pi r} e^{-jkr}\left(\cos\theta\cos\phi\,\hat{a}_\theta - \sin\phi\,\hat{a}_\phi\right)\\ \bar{H}_{FF} &= \frac{j\omega\mu I_0 l}{\eta\, 4\pi r} e^{-jkr}\left(-\sin\phi\,\hat{a}_\theta - \cos\theta\cos\phi\,\hat{a}_\phi\right)\end{aligned}}$$

b) (2-12a) $U(\theta,\phi) = \frac{r^2}{2\eta}\left[|E_\theta|^2 + |E_\phi|^2\right]$

$$U(\theta,\phi) = \frac{\cancel{r^2}}{2\eta}\left[\frac{\omega^2\mu^2|I_0|^2\ell^2}{16\pi^2\cancel{r^2}}\cos^2\theta\cos^2\phi + \frac{\omega^2\mu^2|I_0|^2\ell^2}{16\pi^2\cancel{r^2}}\sin^2\phi\right]$$

$$= \underbrace{\frac{\omega^2\mu^2|I_0|^2\ell^2}{32\eta\pi^2}}_{U_0}\left(\cos^2\theta\cos^2\phi + \sin^2\phi\right)$$

(2-13) $$P_{rad} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} U_0\left(\cos^2\theta\cos^2\phi + \sin^2\phi\right)\sin\theta\, d\theta\, d\phi$$

$$= U_0\int_{\phi=0}^{2\pi}\left[\cos^2\phi\underbrace{\left(\frac{-\cos^3\theta}{3}\right)}_{\to 2/3} + \sin^2\phi\underbrace{(-\cos\theta)}_{\to 2}\right]\Bigg|_{\theta=0}^{\pi} d\phi$$

$$= U_0\int_{\phi=0}^{2\pi}\left(\tfrac{2}{3}\cos^2\phi + 2\sin^2\phi\right)d\phi$$

$$P_{rad} = U_0\left[\tfrac{2}{3}\left(\frac{\phi}{2} + \cancel{\frac{\sin 2\phi}{4}}\right) + 2\left(\frac{\phi}{2} - \cancel{\frac{\sin 2\phi}{4}}\right)\right]\Bigg|_{\phi=0}^{2\pi}$$

$$= U_0\left[\frac{2\pi}{3} + 2\pi\right] = U_0\frac{8\pi}{3}$$

$$D(\theta,\phi) = \frac{4\pi U(\theta,\phi)}{P_{rad}} = \frac{4\pi U_0\left(\cos^2\theta\cos^2\phi + \sin^2\phi\right)}{U_0\, 8\pi/3}$$

$$\underline{\underline{D(\theta,\phi) = \tfrac{3}{2}\left(\cos^2\theta\cos^2\phi + \sin^2\phi\right)}}$$

$$D_0 = 3/2 \quad \left(\begin{array}{l} @\ \phi = \pi/2,\ \text{any } \theta \\ \phi = 3\pi/2,\ \text{any } \theta \end{array}\right)$$

$$\underline{\underline{D_0 = 3/2 = 1.5 = 1.7609\ \text{dBi}}}$$