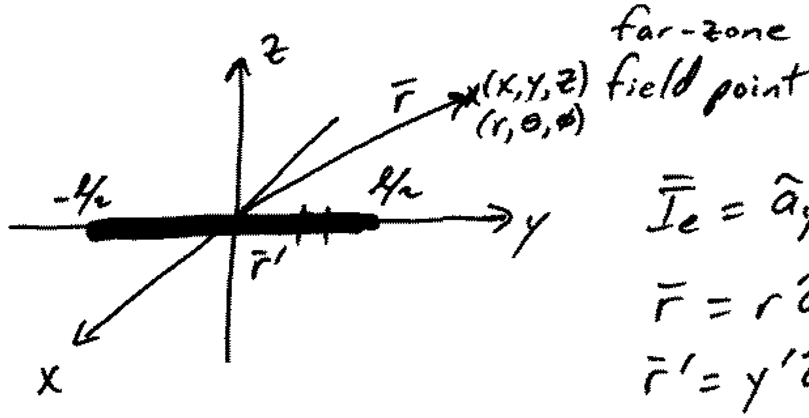
4.2 Repeat Problem 4.1 for a horizontal infinitesimal electric dipole directed along the y-axis.

4.1 A horizontal infinitesimal electric dipole of constant current I_0 is placed symmetrically about the origin and directed along the x-axis. Derive the

(a) far-zone fields radiated by the dipole

(b) directivity of the antenna

- First, find the vector magnetic potential $\bar{A}$ in spherical coordinates.



$$\bar{\bar{I}}_e = \hat{a}_y I_o \qquad d\ell' = dy'$$

$$\bar{r} = r\,\hat{a}_r$$

$$\bar{r}' = y'\hat{a}_y$$

$$R = |\bar{r} - \bar{r}'| \simeq r \quad \left(\begin{array}{c}\text{infinitesimal dipole}\\ r \gg r'\end{array}\right)$$

$$(4\text{-}2) \quad \bar{A} = \frac{\mu}{4\pi}\int \bar{I}_e \frac{e^{-jkR}}{R}\, d\ell'$$

$$= \frac{\mu}{4\pi}\int_{y'=-\ell/2}^{\ell/2} \hat{a}_y I_o \frac{e^{-jkr}}{r}\, dy' = \hat{a}_y \frac{\mu I_o}{4\pi}\frac{e^{-jkr}}{r}\int_{y'=-\ell/2}^{\ell/2} dy'$$

$$= \hat{a}_y \frac{\mu I_o \ell}{4\pi}\frac{e^{-jkr}}{r}$$

use $\hat{a}_y = \sin\theta\sin\phi\,\hat{a}_r + \cos\theta\sin\phi\,\hat{a}_\theta + \cos\phi\,\hat{a}_\phi$ to convert $\bar{A}$ to spherical coordinates

$$\bar{A} = \frac{\mu I_o \ell}{4\pi}\frac{e^{-jkr}}{r}\left(\sin\theta\sin\phi\,\hat{a}_r + \cos\theta\sin\phi\,\hat{a}_\theta + \cos\phi\,\hat{a}_\phi\right)$$

a) To find far-zone fields, use (3-58a) & (3-58b)

(3-58a) $\bar{E} = -j\omega \bar{A}$ (only θ & ϕ components w/ $1/r$ dependence)

$$\bar{E} = \frac{-j\omega\mu I_o l}{4\pi} \frac{e^{-jkr}}{r} \left(\cos\theta \sin\phi \, \hat{a}_\theta + \cos\phi \, \hat{a}_\phi\right)$$

(3-58b) $\bar{H} = -j\frac{\omega}{\eta} \hat{a}_r \times \bar{A}$ (only components w/ $1/r$ dependence)

$$= \frac{-j\omega}{\eta} \hat{a}_r \times \frac{\mu I_o l}{4\pi} \frac{e^{-jkr}}{r} \left(\sin\theta\sin\phi \, \hat{a}_r + \cos\theta\sin\phi \, \hat{a}_\theta + \cos\phi \, \hat{a}_\phi\right)$$

$$= \frac{-j\omega\mu I_o l}{4\pi\eta} \frac{e^{-jkr}}{r} \left(0 + \cos\theta\sin\phi \, \hat{a}_\phi + \cos\phi\,(-\hat{a}_\theta)\right)$$

$$\bar{H} = \frac{-j\omega\mu I_o l}{4\pi\eta} \frac{e^{-jkr}}{r} \left[-\cos\phi \, \hat{a}_\theta + \cos\theta\sin\phi \, \hat{a}_\phi\right]$$

b) To find directivity, first calculate radiation intensity, then power radiated, and finally the directivity.

(2-12a) $U(\theta,\phi) = \frac{r^2}{2\eta}\left[|E_\theta|^2 + |E_\phi|^2\right]$

$$U(\theta,\phi) = \frac{r^2}{2\eta}\left[\frac{\omega^2\mu^2|I_o|^2 l^2}{(4\pi)^2} \frac{1}{r^2}\left(\cos^2\theta\sin^2\phi + \cos^2\phi\right)\right]$$

$$U(\theta,\phi) = \underbrace{\frac{\omega^2\mu^2|I_o|^2 l^2}{32\pi^2\eta}}_{U_o} \left(\cos^2\theta\sin^2\phi + \cos^2\phi\right)$$

b) cont.

(2-13) $$P_{rad} = \oint_\Omega U(\theta,\phi)\, d\Omega = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} U_0\left(\cos^2\theta \sin^2\phi + \cos^2\phi\right)\sin\theta\, d\theta\, d\phi$$

$$= U_0 \int_{\phi=0}^{2\pi}\left[\left(-\frac{\cos^3\theta}{3}\right)\Bigg|_{\theta=0}^{\pi}\sin^2\phi + \left(-\cos\theta\right)\Bigg|_{\theta=0}^{\pi}\cos^2\phi\right]d\phi$$

$$= U_0 \int_{\phi=0}^{2\pi}\left[\left(\frac{+1}{3}+\frac{1}{3}\right)\sin^2\phi + (+1+1)\cos^2\phi\right]d\phi$$

$$= \left(\tfrac{2}{3}\left[\frac{\phi}{2} - \frac{\sin 2\phi}{4}\right]\Bigg|_{\phi=0}^{2\pi} + 2\left[\frac{\phi}{2}+\frac{\sin 2\phi}{4}\right]\Bigg|_{\phi=0}^{2\pi}\right)U_0$$

$$= \left(\tfrac{2}{3}\left[\left(\frac{2\pi}{2}-0\right)-(0-0)\right] + 2\left[\left(\frac{2\pi}{2}+0\right)-(0+0)\right]\right)U_0$$

$$= \left(\tfrac{2}{3}\pi + 2\pi\right)U_0$$

$$\boxed{P_{rad} = \frac{8\pi}{3}U_0 = 8.37758\, U_0 = \frac{8\pi}{3}\frac{\omega^2\mu^2|I_0|^2\ell^2}{32\pi^2\eta} = 0.026526\,\frac{\omega^2\mu^2|I_0|^2\ell^2}{\eta}}$$

(2-16) $$D = \frac{4\pi U}{P_{rad}} = \frac{4\pi U_0\left(\cos^2\theta\sin^2\phi + \cos^2\phi\right)}{8\pi/3\; U_0}$$

$$\underline{\underline{D(\theta,\phi) = \tfrac{3}{2}\left(\cos^2\theta \sin^2\phi + \cos^2\phi\right)}}$$

$\underline{\underline{D_0 = 3/2 = 1.7609\ dB_i}}$ @ $\theta = 0$ or π & $\phi = \pi/2$ or $\frac{3\pi}{2}$

or

$\underbrace{\theta = \text{anything} \ \&\ \phi = 0 \text{ or } \pi}_{\text{x-z plane!}}$