If the vector electric potential for an antenna is $\bar{F} = \hat{a}_z F_0 \frac{e^{-jkr}}{r}$, find $\bar{E} = \bar{E}_{FF}$ and $\bar{H} = \bar{H}_{FF}$ in the **far-field**. Give your answers in spherical coordinates. Assume $\bar{A} = 0$. Factor out common terms, e.g., $F_0 \frac{e^{-jkr}}{r}$.

Method 1 $\Rightarrow$ convert $\hat{a}_z$ to spherical coordinates

$$\bar{F} = (\hat{a}_r \cos\theta - \hat{a}_\theta \sin\theta) F_0 \frac{e^{-jkr}}{r}$$

$$\bar{F} = \hat{a}_r F_0 \cos\theta \frac{e^{-jkr}}{r} - \hat{a}_\theta F_0 \sin\theta \frac{e^{-jkr}}{r}$$

Use (3-59b) to determine $\bar{E}_{FF}$

$$E_r \simeq 0, \; E_\theta \simeq -j\omega\eta \underbrace{F_\phi}_{\to 0} = 0, \text{ and}$$

$$E_\phi \simeq j\omega\eta F_\theta = j\omega\eta \left(-F_0 \sin\theta \frac{e^{-jkr}}{r}\right)$$

putting back into vector form

$$\underline{\underline{\bar{E}_{FF} = -\hat{a}_\phi j\omega\eta F_0 \sin\theta \left(\frac{e^{-jkr}}{r}\right)}} \quad \text{Far-field}$$

Use (3-59a) to determine $\bar{H}_{FF}$

$$H_r \simeq 0, \; H_\theta \simeq -j\omega F_\theta = +j\omega F_0 \sin\theta \frac{e^{-jkr}}{r},$$

$$\text{and } H_\phi \simeq -j\omega \underbrace{F_\phi}_{\to 0} = 0$$

putting back into vector form

$$\underline{\underline{\bar{H}_{FF} = \hat{a}_\theta j\omega F_0 \sin\theta \left(\frac{e^{-jkr}}{r}\right)}} \quad \text{Far-field}$$

Method 2 Use results of computing $\bar{E}$ and $\bar{H}$ everywhere from $\bar{F}$ and drop all terms $\propto \frac{1}{r^n}$ where $n \geq 2$.

From another problem:

$$\bar{E}_F = -\hat{a}_\phi \frac{F_o}{\epsilon} \sin\theta \left(\frac{e^{-jkr}}{r} \right) \left(jk + \cancel{\tfrac{1}{r}} \right)$$

← drop $\frac{1}{r^2}$ term

$$\bar{E}_{FF} = -\hat{a}_\phi \frac{F_o jk}{\epsilon} \sin\theta \left(\frac{e^{-jkr}}{r} \right)$$

Note: $\frac{k}{\epsilon} = \frac{\omega/c}{\epsilon} = \frac{\omega\sqrt{\mu\epsilon}}{\epsilon} = \omega\sqrt{\mu/\epsilon} = \omega\eta$

$$\underline{\underline{\bar{E}_{FF} = -\hat{a}_\phi j\omega\eta F_o \sin\theta \left(\frac{e^{-jkr}}{r} \right)}}$$

← same answer!!

From another problem:

drop entire term $\propto \frac{1}{r^2}$ or $\propto \frac{1}{r^3}$

$$\bar{H}_F = \frac{F_o}{j\omega\mu\epsilon} \left(\frac{e^{-jkr}}{r} \right) \left[\cancel{\frac{2\cos\theta}{r} \left(jk + \frac{1}{r} \right) \hat{a}_r} + \sin\theta \left(-k^2 + \cancel{\frac{jk}{r}} + \cancel{\frac{1}{r^2}} \right) \hat{a}_\theta \right]$$

drop $\frac{1}{r^2}$ + $\frac{1}{r^3}$ terms

$$\bar{H}_{FF} = \hat{a}_\theta \frac{-k^2 F_o \sin\theta}{j\omega\mu\epsilon} \left(\frac{e^{-jkr}}{r} \right)$$

Note: $\frac{k^2}{\omega\mu\epsilon} = \frac{\omega^2/c^2}{\omega/c^2} = \omega$

$$\underline{\underline{\bar{H}_{FF} = \hat{a}_\theta j\omega F_o \sin\theta \left(\frac{e^{-jkr}}{r} \right)}}$$

← same answer!!