If the vector <u>electric</u> potential for an antenna is $\overline{F} = \hat{a}_y C_0 \dfrac{e^{-jkr}}{r}$, find $\overline{E} = \overline{E}_{FF}$ and $\overline{H} = \overline{H}_{FF}$ in the **far-field**. Give your answers in spherical coordinates. Assume $\overline{A} = 0$.

<u>Method 1</u> There are no $\frac{1}{r^n}$ terms w/ $n \geq 2$ in $\overline{F}$.

Use (3-59b) to determine $\overline{E}_{FF}$

$$E_r \simeq 0, \quad E_\theta \simeq -j\omega\eta F_\phi, \text{ and } E_\phi \simeq j\omega\eta F_\theta$$

Converting the unit vector $\hat{a}_y$ to spherical coordinates–

$$\overline{F} = C_0 \frac{e^{-jkr}}{r}\left[\sin\theta\sin\phi\,\hat{a}_r + \cos\theta\sin\phi\,\hat{a}_\theta + \cos\phi\,\hat{a}_\phi\right]$$

which allows us to write:

$$E_\theta \simeq -j\omega\eta\, C_0 \frac{e^{-jkr}}{r}\cos\phi$$

$$E_\phi \simeq j\omega\eta\, C_0 \frac{e^{-jkr}}{r}\cos\theta\sin\phi$$

Putting components back in vector form:

$$\underline{\underline{\overline{E}_{FF} \cong j\omega\eta\, C_0 \frac{e^{-jkr}}{r}\left[-\cos\phi\,\hat{a}_\theta + \cos\theta\sin\phi\,\hat{a}_\phi\right]}}$$

Use (3-59a) to find $\overline{H}_{FF}$

$$H_r \simeq 0, \quad H_\theta \simeq -j\omega F_\theta, \text{ and } H_\phi \simeq -j\omega F_\phi$$

So $H_\theta \simeq -j\omega\, C_0 \dfrac{e^{-jkr}}{r}\cos\theta\sin\phi$

$$H_\phi \simeq -j\omega\, C_0 \frac{e^{-jkr}}{r}\cos\phi$$

which in vector form is:

$$\underline{\underline{\overline{H}_{FF} = -j\omega\, C_0 \frac{e^{-jkr}}{r}\left[\cos\theta\sin\phi\,\hat{a}_\theta + \cos\phi\,\hat{a}_\phi\right]}}$$

Method 2 Use results of calculating $\bar{E}$ & $\bar{H}$ everywhere from $\bar{F}$ and drop all terms $\propto \frac{1}{r^n}$ where $n \geq 2$.

From (3-16), $\bar{E} = \bar{E}_F = \frac{-1}{\epsilon} \bar{\nabla} \times \bar{F}$

drop this term

$$= \frac{C_0}{\epsilon} \frac{e^{-jkr}}{r} \left(jk + \cancel{\tfrac{1}{r}} \right) \left[-\cos\phi\, \hat{a}_\theta + \cos\theta \sin\phi\, \hat{a}_\phi \right]$$

which yields

$$\bar{E}_{FF} = \frac{jk C_0}{\epsilon} \frac{e^{-jkr}}{r} \left[-\cos\phi\, \hat{a}_\theta + \cos\theta \sin\phi\, \hat{a}_\phi \right]$$

Note: $\frac{k}{\epsilon} = \frac{2\pi/\lambda}{\epsilon} = \frac{2\pi/(c/f)}{\epsilon} = \frac{\omega/c}{\epsilon} = \frac{\omega\sqrt{\mu\epsilon}}{\epsilon} = \omega\sqrt{\frac{\mu}{\epsilon}} = \omega\eta$

$$\underline{\underline{\bar{E}_{FF} = j\omega\eta C_0 \frac{e^{-jkr}}{r} \left[-\cos\phi\, \hat{a}_\theta + \cos\theta \sin\phi\, \hat{a}_\phi \right]}}$$ Same result!

From (3-21), $\bar{H} = \bar{H}_F = \frac{\bar{\nabla} \times \bar{E}_F}{-j\omega\mu}$

drop

$$= \frac{C_0}{j\omega\mu\epsilon} \frac{e^{-jkr}}{r} \Big[2\sin\theta \sin\phi \left(\cancel{\tfrac{jk}{r}} + \cancel{\tfrac{1}{r^2}} \right) \hat{a}_r$$

$$+ \cos\theta \sin\phi \left(k^2 - \cancel{\tfrac{jk}{r}} - \cancel{\tfrac{1}{r^2}} \right) \hat{a}_\theta$$

$$+ \cos\phi \left(k^2 - \cancel{\tfrac{jk}{r}} - \cancel{\tfrac{1}{r^2}} \right) \hat{a}_\phi \Big]$$

which yields

$$\bar{H}_{FF} = \frac{C_0 k^2}{j\omega\mu\epsilon} \frac{e^{-jkr}}{r} \left[\cos\theta \sin\phi\, \hat{a}_\theta + \cos\phi\, \hat{a}_\phi \right]$$

where $k^2 = \frac{\omega^2}{c^2} = \omega^2 \mu\epsilon$

$$\underline{\underline{\bar{H}_{FF} = -j\omega C_0 \frac{e^{-jkr}}{r} \left[\cos\theta \sin\phi\, \hat{a}_\theta + \cos\phi\, \hat{a}_\phi \right]}}$$ Same result!