Given that the vector <u>magnetic</u> potential for an antenna is $\bar{A} = \hat{a}_\theta A_0 \cos\theta \left[\frac{e^{-jkr}}{r} + \frac{jke^{-jkr}}{r^2} \right]$, find $\bar{E} = \bar{E}_{FF}$ and $\bar{H} = \bar{H}_{FF}$ in the **far-field**. Give your answers in spherical coordinates. Assume $\bar{F} = 0$. Factor out common terms, e.g., $A_0 \frac{e^{-jkr}}{r}$.

<u>Method 1</u> Per section 3.6 of Balanis text, drop terms of $\bar{A} \propto \frac{1}{r^n}$ where $n \geq 2$

$$\bar{A}_{FF} \simeq \hat{a}_\theta A_0 \cos\theta \left[\frac{e^{-jkr}}{r} + \underbrace{\frac{jke^{-jkr}}{r^2}}_{\to 0} \right]$$

Then, using (3-58a) $\bar{E}_{FF} = \bar{E}_A \simeq -j\omega \bar{A}_{FF}$

$$\bar{E}_{FF} = -\hat{a}_\theta \, j\omega A_0 \cos\theta \, \frac{e^{-jkr}}{r}$$

Using (3-58b), the far-zone magnetic field components are:

$$H_r \simeq 0 \,, \quad H_\theta \simeq j\frac{\omega}{\eta} \underbrace{A_\phi}_{\to 0} \,, \quad \text{and} \quad H_\phi \simeq -j\frac{\omega}{\eta} A_\theta$$

$$\bar{H}_{FF} = \hat{a}_\phi H_\phi = -\hat{a}_\phi \frac{j\omega A_0}{\eta} \cos\theta \, \frac{e^{-jkr}}{r}$$

Method 2 Compute $\bar{E}$ + $\bar{H}$ due to $\bar{A}$ everywhere.
Then, drop all $\bar{E}$ + $\bar{H}$ terms $\propto \frac{1}{r^n}$ $n \geq 2$.

From (3-2a), $\bar{H} = \bar{H}_A = \frac{1}{\mu} \bar{\nabla} \times \bar{A}$

drop these terms

$$= -\hat{a}_\phi \frac{jkA_o}{\mu} \cos\theta \frac{e^{-jkr}}{r} \left(1 + \frac{jk}{r} + \frac{1}{r^2}\right)$$

$$\bar{H}_{FF} = -\hat{a}_\phi \frac{jkA_o}{\mu} \cos\theta \frac{e^{-jkr}}{r}$$

Note: $\frac{k}{\mu} = \frac{\omega/c}{\mu} = \frac{\omega\sqrt{\mu\epsilon}}{\mu} = \frac{\omega}{\sqrt{\mu/\epsilon}} = \omega/\eta$

$$\underline{\underline{\bar{H}_{FF} = -\hat{a}_\phi \frac{j\omega A_o}{\eta} \cos\theta \frac{e^{-jkr}}{r}}}$$ ← same answer

From (3-10), $\bar{E} = \bar{E}_A = \frac{\bar{\nabla} \times \bar{H}_A}{j\omega\epsilon}$

drop entire $\hat{a}_r$ term

$$= -\hat{a}_r 2A_o c \frac{\cos 2\theta}{\sin\theta} \frac{e^{-jkr}}{r^2} \left(1 + \frac{jk}{r} + \frac{1}{r^2}\right)$$

$$+ \hat{a}_\theta A_o c \cos\theta \frac{e^{-jkr}}{r} \left(-jk + \underbrace{\frac{k^2}{r} - \frac{2jk}{r^2} - \frac{2}{r^3}}_{\text{drop these terms}}\right)$$

$$\bar{E}_{FF} = -\hat{a}_\theta jkc A_o \cos\theta \frac{e^{-jkr}}{r}$$

Note: $kc = \frac{\omega}{c}(c) = \omega$

$$\underline{\underline{\bar{E}_{FF} = -\hat{a}_\theta j\omega A_o \cos\theta \frac{e^{-jkr}}{r}}}$$ ← same answer