Given the vector magnetic potential for an antenna is $\overline{A} = \hat{a}_\phi A_0 \cos\theta \left[\frac{e^{-jkr}}{r} + \frac{j2ke^{-jkr}}{r^2} \right]$, find $\overline{E} = \overline{E}_{FF}$ and $\overline{H} = \overline{H}_{FF}$ in the **far-field**. Give your answers in spherical coordinates. Assume $\overline{F} = 0$. Factor out common terms, e.g., $A_0 \, {e^{-jkr}}/{r}$.

Method 1

First, drop any terms $\propto \frac{1}{r^n}$ for $n \geq 2$ per section 3.6

$$\overline{A} = \hat{a}_\phi A_0 \cos\theta \frac{e^{-jkr}}{r}$$

Then, using (3-58a)

$$\overline{E} = \overline{E}_A = \overline{E}_{FF} = -j\omega \overline{A} \leftarrow \text{only } \theta \,\&\, \phi \text{ components}$$

$$\boxed{\overline{E}_{FF} = -\hat{a}_\phi j\omega A_0 \cos\theta \frac{e^{-jkr}}{r}}$$

Using (3-58b), we can get the far-zone magnetic field

$$H_r \simeq 0$$

$$H_\theta \simeq j\frac{\omega}{\eta} A_\phi = \frac{j\omega A_0 \cos\theta}{\eta} \frac{e^{-jkr}}{r}$$

$$H_\phi \simeq -j\frac{\omega}{\eta} \cancel{A_\theta}^{\,0}$$

$$\boxed{\overline{H}_{FF} = \hat{a}_\theta \frac{j\omega A_0}{\eta} \cos\theta \frac{e^{-jkr}}{r}}$$

Method 2

→ Use prior results that are valid for all r.

$$\bar{E} = -\hat{a}_\phi \, j\omega A_0 \cos\theta \, \frac{e^{-jkr}}{r}\left[1 + \frac{j2k}{r}\right]$$

$$\bar{H} = \hat{a}_r \frac{A_0 \cos 2\theta}{\mu \sin\theta} \frac{e^{-jkr}}{r^2}\left(1 + j2k/r\right)$$

$$+ \hat{a}_\theta \frac{jkA_0 \cos\theta}{\mu} \frac{e^{-jkr}}{r}\left(1 + \frac{j2k}{r} + \frac{2}{r^2}\right)$$

→ Drop all terms $\propto \frac{1}{r^2}, \frac{1}{r^3}, \ldots$ to get $\bar{E}_{FF}$ + $\bar{H}_{FF}$

$$\underline{\underline{\bar{E}_{FF} = -\hat{a}_\phi \, j\omega A_0 \cos\theta \, \frac{e^{-jkr}}{r}}} \leftarrow \text{same!}$$

$$\bar{H}_{FF} = \hat{a}_r(0) + \hat{a}_\theta \frac{jkA_0 \cos\theta}{\mu} \frac{e^{-jkr}}{r}\left(1 + 0 + 0\right)$$

$$\boxed{\begin{aligned}\bar{H}_{FF} &= \hat{a}_\theta \frac{jkA_0 \cos\theta}{\mu} \frac{e^{-jkr}}{r} \\ \bar{H}_{FF} &= \hat{a}_\theta \frac{j\omega A_0}{\eta} \cos\theta \, \frac{e^{-jkr}}{r}\end{aligned}} \leftarrow \text{same!}$$

Note: $\frac{k}{\mu} = \frac{\omega\sqrt{\mu\epsilon}}{\mu} = \frac{\omega}{\sqrt{\mu/\epsilon}} = \omega/\eta$