Given that the vector <u>magnetic</u> potential for an antenna is $\bar{A} = \hat{a}_\theta A_0 \cos\theta \left[\dfrac{e^{-jkr}}{r} + \dfrac{jke^{-jkr}}{r^2}\right]$, find $\bar{E}$ and $\bar{H}$ **everywhere**. Give your answers in spherical coordinates. Assume $\bar{F} = 0$. Factor out common terms, e.g., $A_0 \dfrac{e^{-jkr}}{r}$.

Use (3-2a) $\bar{H} = \bar{H}_A = \frac{1}{\mu}\bar{\nabla}\times\bar{A}$ where $\bar{A} = \hat{a}_\theta A_\theta$

$$\bar{H} = \frac{1}{\mu}\left\{\frac{1}{r\sin\theta}\left[\frac{\partial(\sin\theta\, \cancel{A_\phi}^{\,0})}{\partial\theta} - \frac{\partial A_\theta}{\partial\phi}\right]\hat{a}_r + \left[\frac{1}{r\sin\theta}\frac{\partial \cancel{A_r}^{\,0}}{\partial\phi} - \frac{1}{r}\frac{\partial(r\,\cancel{A_\phi}^{\,0})}{\partial r}\right]\hat{a}_\theta + \frac{1}{r}\left[\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial \cancel{A_r}^{\,0}}{\partial\theta}\right]\hat{a}_\phi\right\}$$

$$= \frac{-1}{\mu r\sin\theta}\underbrace{\frac{\partial\left(A_0\cos\theta\left[\frac{e^{-jkr}}{r} + \frac{jke^{-jkr}}{r^2}\right]\right)}{\partial\phi}}_{\to 0}\hat{a}_r + \frac{1}{\mu r}\frac{\partial}{\partial r}\left(A_0\cos\theta\left[e^{-jkr} + \frac{jke^{-jkr}}{r}\right]\right)\hat{a}_\phi$$

$$= \frac{A_0\cos\theta}{\mu r}\left[(-jk)e^{-jkr} + jk(-jk)\frac{e^{-jkr}}{r} - \frac{jke^{-jkr}}{r^2}\right]\hat{a}_\phi$$

$$\underline{\underline{\bar{H} = -\hat{a}_\phi \frac{jkA_0}{\mu}\cos\theta\, \frac{e^{-jkr}}{r}\left(1 + \frac{jk}{r} + \frac{1}{r^2}\right)}}$$

To determine $\bar{E} = \bar{E}_A$, we can use (3-10) $\bar{\nabla}\times\bar{H}_A = \bar{J} + j\omega\epsilon\bar{E}_A$ or (3-15) $\bar{E} = \bar{E}_A = -j\omega\bar{A} - j\frac{1}{\omega\mu\epsilon}\bar{\nabla}(\bar{\nabla}\cdot\bar{A})$. In this case, (3-10) w/ $\bar{J} = 0$ appears to be more straight forward.

$\bar{E}_A = \frac{1}{j\omega\epsilon}\bar{\nabla}\times\bar{H}_A$ where $\bar{H}_A = \hat{a}_\phi H_\phi$ & $\bar{J} = 0$

$$= \frac{1}{j\omega\epsilon}\left\{\frac{1}{r\sin\theta}\left[\frac{\partial(\sin\theta H_\phi)}{\partial\theta} - \frac{\partial \cancel{H_\theta}^{\,0}}{\partial\phi}\right]\hat{a}_r + \left[\frac{1}{r\sin\theta}\frac{\partial \cancel{H_r}^{\,0}}{\partial\phi} - \frac{1}{r}\frac{\partial(rH_\phi)}{\partial r}\right]\hat{a}_\theta + \frac{1}{r}\left[\frac{\partial(r\cancel{H_\theta}^{\,0})}{\partial r} - \frac{\partial \cancel{H_r}^{\,0}}{\partial\theta}\right]\hat{a}_\phi\right\}$$

$$\bar{E} = \bar{E}_A = \frac{\hat{a}_r}{j\omega\epsilon r\sin\theta}\frac{\partial}{\partial\theta}\left(\frac{-jkA_0}{\mu}\cos\theta\sin\theta\,\frac{e^{-jkr}}{r}\left(1+\frac{jk}{r}+\frac{1}{r^2}\right)\right)$$

$$+\frac{+1\hat{a}_\theta}{j\omega\epsilon r}\frac{\partial}{\partial r}\left(\frac{+jkA_0}{\mu}\cos\theta\, e^{-jkr}\left(1+\frac{jk}{r}+\frac{1}{r^2}\right)\right)$$

$$= \frac{\hat{a}_r}{\cancel{j}\omega\epsilon r\sin\theta}\,\frac{-\cancel{j}kA_0}{\mu}\left(\overset{\cos 2\theta}{\cancel{\cos^2\theta-\sin^2\theta}}\right)\frac{e^{-jkr}}{r}\left(1+\frac{jk}{r}+\frac{1}{r^2}\right)$$

$$+\frac{\hat{a}_\theta\,\cancel{j}kA_0\cos\theta}{\cancel{j}\omega\epsilon r\mu}\left[(-jk)e^{-jkr}\left(1+\frac{jk}{r}+\frac{1}{r^2}\right)+e^{-jkr}\left(0-\frac{jk}{r^2}-\frac{2}{r^3}\right)\right]$$

$$\bar{E} = -\hat{a}_r\,\frac{2A_0k}{\omega\epsilon\mu}\,\frac{\cos 2\theta}{\sin\theta}\,\frac{e^{-jkr}}{r^2}\left(1+\frac{jk}{r}+\frac{1}{r^2}\right)$$

$$+\hat{a}_\theta\,\frac{A_0k}{\omega\epsilon\mu}\cos\theta\,\frac{e^{-jkr}}{r}\left(-jk+\frac{k^2}{r}-\frac{2jk}{r^2}-\frac{2}{r^3}\right)$$

Note: $\dfrac{k}{\omega\mu\epsilon} = \dfrac{\omega/c}{\omega\mu\epsilon} = \dfrac{\omega\sqrt{\mu\epsilon}}{\omega\mu\epsilon} = \dfrac{1}{\sqrt{\mu\epsilon}} = c$

$$\bar{E} = -\hat{a}_r\,2A_0c\,\frac{\cos 2\theta}{\sin\theta}\,\frac{e^{-jkr}}{r^2}\left(1+\frac{jk}{r}+\frac{1}{r^2}\right)$$

$$+\hat{a}_\theta A_0c\cos\theta\,\frac{e^{-jkr}}{r}\left(-jk+\frac{k^2}{r}-\frac{j2k}{r^2}-\frac{2}{r^3}\right)$$