Given the vector <u>magnetic</u> potential for an antenna is $\bar{A} = \hat{a}_\phi A_0 \cos\theta \left[\frac{e^{-jkr}}{r} + \frac{j2ke^{-jkr}}{r^2} \right]$, find $\bar{E}$ and $\bar{H}$ **everywhere**. Give your answers in spherical coordinates. Assume $\bar{F} = 0$. Factor out common terms, e.g., $A_0 \, e^{-jkr}/r$.

Per (3-2a), $\bar{H}_A = \frac{1}{\mu} \bar{\nabla} \times \bar{A}$ where $\bar{A} = \hat{a}_\phi A_\phi$

Since $\bar{F} = 0$, $\bar{H} = \bar{H}_A + \cancel{\bar{H}_F}^{\,0} = \bar{H}_A$.

$$\bar{H} = \frac{1}{\mu} \left\{ \frac{1}{r\sin\theta} \left[\frac{\partial(\sin\theta A_\phi)}{\partial\theta} - \frac{\partial \cancel{A_\theta}^{\,0}}{\partial\phi} \right] \hat{a}_r + \left[\frac{1}{r\sin\theta} \frac{\partial \cancel{A_r}^{\,0}}{\partial\phi} - \frac{1}{r} \frac{\partial(rA_\phi)}{\partial r} \right] \hat{a}_\theta \right.$$
$$\left. + \frac{1}{r} \left[\frac{\partial(r \cancel{A_\theta}^{\,0})}{\partial r} - \frac{\partial \cancel{A_r}^{\,0}}{\partial\theta} \right] \hat{a}_\phi \right\}$$

$$= \frac{1}{\mu r \sin\theta} \frac{\partial}{\partial\theta} \left(A_0 \cos\theta \sin\theta \left[\frac{e^{-jkr}}{r} + \frac{j2ke^{-jkr}}{r^2} \right] \right) \hat{a}_r$$
$$- \frac{1}{\mu r} \frac{\partial}{\partial r} \left(A_0 \cos\theta \left[e^{-jkr} + \frac{j2ke^{-jkr}}{r} \right] \right) \hat{a}_\theta$$

$$= \frac{1}{\mu r \sin\theta} A_0 \left(\cos^2\theta - \sin^2\theta \right) \left[\frac{e^{-jkr}}{r} + \frac{j2ke^{-jkr}}{r^2} \right] \hat{a}_r$$
$$- \frac{1}{\mu r} A_0 \cos\theta \left[-jk\, e^{-jkr} - \frac{j2ke^{-jkr}}{r^2} + \frac{j2k(-jk)e^{-jkr}}{r} \right] \hat{a}_\theta$$

Factor out common terms & use $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$$\boxed{\bar{H} = \frac{A_0}{\mu} \frac{e^{-jkr}}{r} \left[\hat{a}_r \frac{\cos 2\theta}{r\sin\theta} \left(1 + \frac{j2k}{r} \right) + \hat{a}_\theta\, jk\cos\theta \left(1 + \frac{j2k}{r} + \frac{2}{r^2} \right) \right]}$$

To find $\bar{E} = \bar{E}_A$, use (3-15) $\bar{E}_A = -j\omega\bar{A} - j\frac{1}{\omega\mu\epsilon}\bar{\nabla}(\bar{\nabla}\cdot\bar{A})$

$$\bar{\nabla}\cdot\bar{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \cancelto{0}{A_r}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\cancelto{0}{A_\theta}\sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$= \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left(A_0\cos\theta\left[\frac{e^{-jkr}}{r} + \frac{j2k e^{-jkr}}{r^2}\right]\right)$$

$\underline{\bar{\nabla}\cdot\bar{A} = 0}$ (No ϕ dependence in A_ϕ)

$$\bar{E} = \bar{E}_A = -j\omega\bar{A}$$

$$\bar{E} = \hat{a}_\phi - j\omega A_0\cos\theta\left[\frac{e^{-jkr}}{r} + \frac{j2k e^{-jkr}}{r^2}\right]$$

$$\underline{\underline{\bar{E} = -\hat{a}_\phi\, j\omega A_0\cos\theta\,\frac{e^{-jkr}}{r}\left(1 + \frac{j2k}{r}\right)}}$$