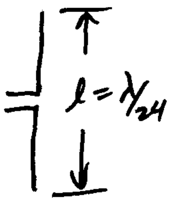
In free space, a small dipole of length λ/24 has an electric field given by (4-36a). Find a) the vector effective length, b) maximum value of the vector effective length and angle(s) at which it occurs, c) the ratio of the maximum effective length to the physical length, and d) the open-circuit voltage if a vertically-polarized uniform plane wave impinges on the dipole with a power density of 0.4 mW/m^2 at 2.4 GHz at broadside.



(4-36a) $E_\theta = j\eta \frac{k I_0 l e^{-jkr}}{8\pi r}\sin\theta$ $(kr >> 1)$ far-field

$k = 2\pi/\lambda$ + $l = \lambda/24$ $\quad 0 \le \theta \le 180^\circ$

a) Per (2-92), $\bar{E}_a = \hat{a}_\theta E_\theta + \hat{a}_\phi \cancel{E_\phi}^{\,0} = -j\eta \frac{k I_{in}}{4\pi r}\bar{l}_e e^{-jkr}$

$$\hat{a}_\theta j\eta \frac{k I_0 l e^{-jkr}}{8\pi r}\sin\theta = -j\eta\frac{k I_{in}}{4\pi r}\bar{l}_e e^{-jkr}$$

Assume $I_0 = I_{in}$ and solve for $\bar{l}_e$

$$\bar{l}_e = \hat{a}_\theta \cancel{j\eta}\frac{\cancel{k I_0}\, l\, \cancel{e^{-jkr}}}{8\cancel{\pi r}}\sin\theta\,\frac{4\cancel{\pi r}}{-\cancel{j\eta}\,\cancel{k I_{in}}}\cancel{e^{jkr}}$$

$$\bar{l}_e = -\hat{a}_\theta\frac{l}{2}\sin\theta = -\hat{a}_\theta\frac{\lambda}{48}\sin\theta \qquad 0\le\theta\le180^\circ$$

b) $l_{e,max} = |\bar{l}_{e,max}| = l/2 = \lambda/48$ @ $\theta = 90^\circ$

c) $\frac{l_{e,max}}{l} = \frac{\lambda/48}{\lambda/24} = 0.5$

d) Per (2-93), $V_{oc} = \bar{E}^i\cdot\bar{l}_e$ @ origin $\bar{E}_i = \hat{a}_z E_{i,0}e^{-jk(0)}$

where $W_0 = |E_{i,0}|^2/2\eta_0 \Rightarrow E_{i,0} = \sqrt{2\eta_0 W_0} = \sqrt{2(376.73)0.4\times10^{-3}}$

$= 0.54898$ V/m

$$V_{oc} = \hat{a}_z 0.54898 \cdot -\overset{\hat{a}_z}{\cancel{\hat{a}_\theta}}\,\lambda/48\,\sin 90^\circ = +(1)\,0.54898\left(\frac{1}{48}\right)\frac{2.9979\times10^8}{2.4\times10^9}$$

$V_{oc} = 0.001429\ \text{V} = 1.4286\ \text{mV}$