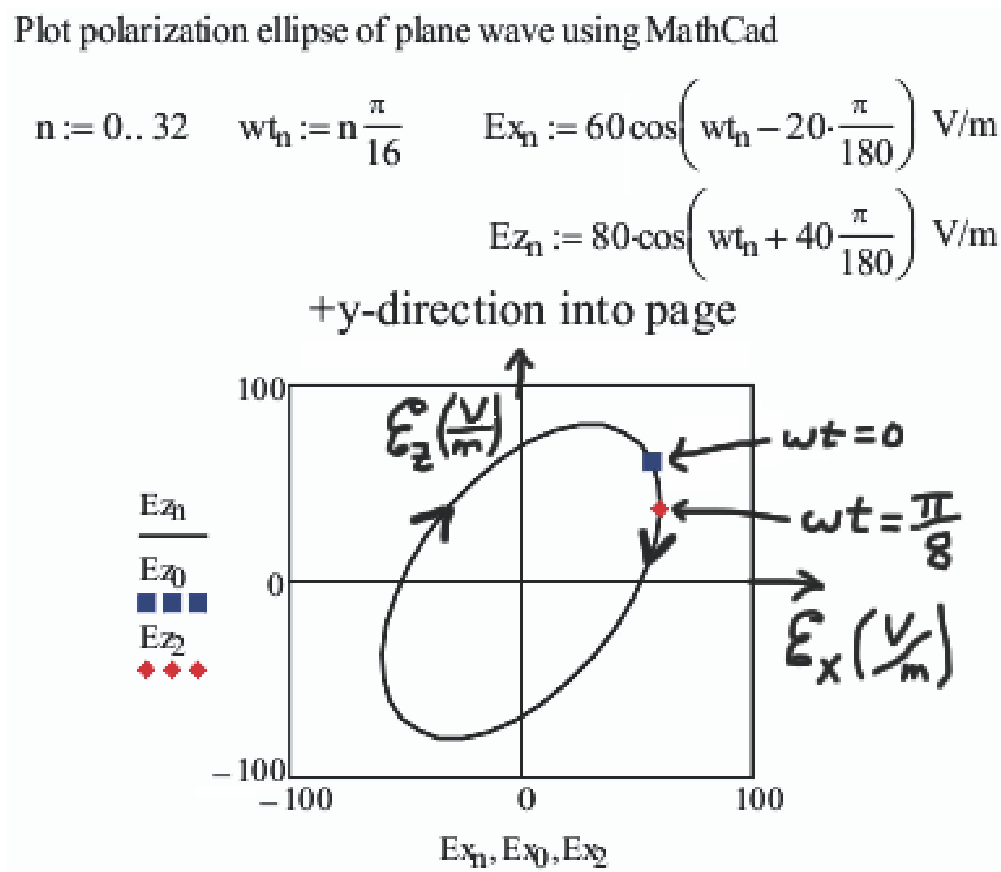
A plane wave $\bar{\mathcal{E}}_i = \hat{a}_x 60\cos(\omega t - 20^\circ - \beta y) + \hat{a}_z 80\cos(\omega t + 40^\circ - \beta y)$ (V/m) is incident on an infinitesimal dipole located at the origin and oriented along the *z*-axis. What direction is the plane wave traveling? Sketch the polarization ellipse of the incident plane wave w/ wave propagating into page and annotate with its polarization. Next, find $\hat{\rho}_w$, $\hat{\rho}_a$, and the PLF when $\theta = 90°$ and $\phi = 270°$. [Hints: Look at Chapter 4 section on infinitesimal dipoles and remember how to convert from spherical to Cartesian unit vectors.]

- From the '-βy' term, the **plane wave is traveling in the +*y*-direction**.
- At $y = 0$, $\bar{\mathcal{E}}_i(y=0,t) = \hat{a}_x 60\cos(\omega t - 20^\circ) + \hat{a}_z 80\cos(\omega t + 40^\circ)$ (V/m)



- From plot, the plane wave has **RH elliptical polarization**.
- At $y = 0$, the phasor electric field of the incident plane wave is-

$$\bar{E}_i(y=0) = \bar{E}_i = \hat{a}_x 60\angle -20^\circ + \hat{a}_z 80\angle 40^\circ \text{ (V/m)}$$

- Per (2-69), $\bar{E}_i = \hat{a}_x 60\angle -20^\circ + \hat{a}_z 80\angle 40^\circ \text{ (V/m)} = \hat{\rho}_w E_i \quad \Rightarrow \quad \hat{\rho}_w = \bar{E}_i / E_i$
- $E_i = \sqrt{\bar{E}_i \bullet \bar{E}_i^*} = \sqrt{60^2 + 80^2} = 100$ (V/m)
- $\hat{\rho}_w = \hat{a}_x 60\angle -20^\circ + \hat{a}_z 80\angle 40^\circ / 100 \quad \Rightarrow \quad \underline{\underline{\hat{\rho}_w = \hat{a}_x 0.6\angle -20^\circ + \hat{a}_z 0.8\angle 40^\circ}}$

From Chapter 4, the far-field electric field for an infinitesimal dipole is given by (4-26a)

$$E_\theta = j\eta \frac{kI_0 \ell e^{-jkr}}{4\pi r}\sin\theta \Rightarrow \bar{E}_\theta = \hat{a}_\theta j\eta \frac{kI_0\ell e^{-jkr}}{4\pi r}\sin\theta$$

@ $\theta = 90^\circ$ and $\phi = 270^\circ$

$$\bar{E}_a = \bar{E}_\theta = \hat{a}_\theta j\eta \frac{kI_0 \ell e^{-jkr}}{4\pi r}\overbrace{\sin 90^\circ}^{1} = \hat{\rho}_a E_a \quad (2\text{-}70)$$

$$|\bar{E}_a|^2 = \bar{E}_a \cdot \bar{E}_a^* = \frac{\eta^2 k^2 I_0^2 \ell^2}{(4\pi r)^2} \Rightarrow E_a = \frac{\eta k I_0 \ell}{4\pi r}$$

$$\text{so, } \hat{\rho}_a = \frac{\bar{E}_a}{E_a} = \hat{a}_\theta j\, e^{-jkr} = \hat{a}_\theta e^{j\pi/2} e^{-jkr}$$

$$\hat{\rho}_a = \hat{a}_\theta 1 \angle \pi/2 - kr \quad \text{(assumed } I_0 \text{ was real.)}$$

Next, convert $\hat{a}_\theta$ to cartesian coordinates @ $\theta = 90^\circ$, $\phi = 270^\circ$

$$\hat{a}_\theta = \cos\theta\cos\phi\,\hat{a}_x + \cos\theta\sin\phi\,\hat{a}_y - \sin\theta\,\hat{a}_z$$

$$= \underbrace{\cos 90^\circ}_{\to 0}\cos 270^\circ\,\hat{a}_x + \underbrace{\cos 90^\circ}_{\to 0}\sin 270^\circ\,\hat{a}_y - \sin 90^\circ\,\hat{a}_z = -\hat{a}_z$$

$$\text{So, } \underline{\underline{\hat{\rho}_a = \hat{a}_\theta 1 \angle \pi/2 - kr = -\hat{a}_z 1 \angle \pi/2 - kr}}$$

$$\hat{\rho}_w \cdot \hat{\rho}_a = \left(\hat{a}_x 0.6 \angle -20^\circ + \hat{a}_z 0.8 \angle 40^\circ\right) \cdot -\hat{a}_z 1 \angle \pi/2 - kr$$

$$= -0.8(1) \angle \pi/2 - kr + 40^\circ = -0.8\, e^{j\pi/2} e^{-jkr} e^{j40^\circ}$$

$$\text{Per (2-71), } PLF = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = (\hat{\rho}_w \cdot \hat{\rho}_a)(\hat{\rho}_w \cdot \hat{\rho}_a)^*$$

$$= \left(-0.8\, e^{j\pi/2} e^{-jkr} e^{j40^\circ}\right)\left(-0.8\, e^{-j\pi/2} e^{+jkr} e^{-j40^\circ}\right)$$

$$\underline{\underline{PLF = 0.64 = 10\log_{10} 0.64 = -1.938\ \text{dB}}}$$