A plane wave $\bar{\mathscr{E}} = \hat{a}_x 80\cos(\omega t + 20^\circ - \beta y) + \hat{a}_z 60\cos(\omega t - 40^\circ - \beta y)$ (V/m) is incident on an infinitesimal dipole located at the origin and oriented along the z-axis. What direction is the plane wave traveling? Sketch the polarization ellipse of the plane wave w/ wave propagating into page and annotate with its polarization. What are $\hat{\rho}_w$, $\hat{\rho}_a$, and the PLF when $\theta = 90^\circ$ and $\phi = 30^\circ$? [Hints: Look at Chapter 4 and remember how to convert from spherical to Cartesian unit vectors.]

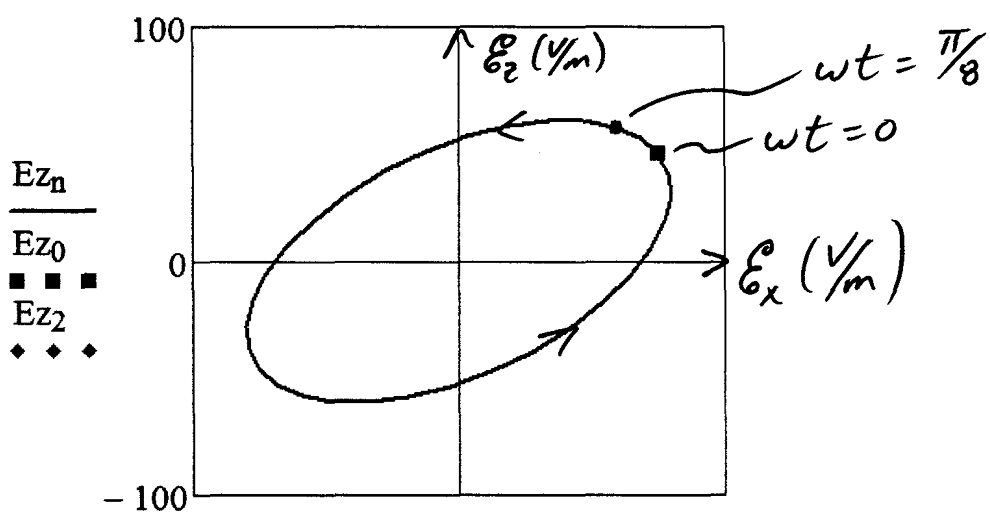
- From the '-βy' term, the **<u>plane wave is traveling in the +y-direction</u>**.
- At $y = 0$, $\bar{\mathscr{E}} = \hat{a}_x 80\cos(\omega t + 20^\circ) + \hat{a}_z 60\cos(\omega t - 40^\circ)$ (V/m)

<u>Plot polarization ellipse of plane wave using MathCad</u>

$n := 0..32 \qquad wt_n := n\frac{\pi}{16} \qquad Ex_n := 80\cos\left(wt_n + 20\cdot\frac{\pi}{180}\right)$

$Ez_n := 60\cdot\cos\left(wt_n - 40\frac{\pi}{180}\right)$

+y-direction into page



∗ <u>LH elliptical polarization</u>

Phasor plane wave @ $y=0$ $\quad \bar{E}_i = \hat{a}_x 80\angle 20^\circ + \hat{a}_z 60\angle -40^\circ$ V/m

$E_i = |\bar{E}_i| = \sqrt{80^2 + 60^2} = 100$ V/m

Per (2-69), $\bar{E}_i = \hat{\rho}_w E_i \Rightarrow \underline{\hat{\rho}_w = \hat{a}_x 0.8\angle 20^\circ + \hat{a}_z 0.6\angle -40^\circ}$

From Chapter 4, the far-field electric field for an infinitesimal dipole is given by (4-26a)

$$E_\theta = j\eta \frac{kI_0 \ell e^{-jkr}}{4\pi r}\sin\theta \Rightarrow \bar{E}_\theta = \hat{a}_\theta j\eta \frac{kI_0 \ell e^{-jkr}}{4\pi r}\sin\theta$$

@ $\theta = 90°$ and $\phi = 30°$

$$\bar{E}_a = \bar{E}_\theta = \hat{a}_\theta j\eta \frac{kI_0 \ell e^{-jkr}}{4\pi r}\underbrace{\sin 90°}_{1} = \hat{\rho}_a E_a \quad (2\text{-}70)$$

$$|\bar{E}_a|^2 = \bar{E}_a \cdot \bar{E}_a^* = \frac{\eta^2 k^2 I_0^2 \ell^2}{(4\pi r)^2} \Rightarrow E_a = \frac{\eta k I_0 \ell}{4\pi r}$$

$$\text{so, } \hat{\rho}_a = \frac{\bar{E}_a}{E_a} = \hat{a}_\theta j e^{-jkr} = \hat{a}_\theta e^{j\pi/2} e^{-jkr}$$

$$\hat{\rho}_a = \hat{a}_\theta 1 \angle \pi/2 - kr \quad \text{(assumed } I_0 \text{ was real)}$$

Next, convert $\hat{a}_\theta$ to cartesian coordinates @ $\theta = 90°$, $\phi = 30°$

$$\hat{a}_\theta = \cos\theta\cos\phi\,\hat{a}_x + \cos\theta\sin\phi\,\hat{a}_y - \sin\theta\,\hat{a}_z$$

$$= \underbrace{\cos 90°}_{0}\cos 30°\,\hat{a}_x + \underbrace{\cos 90°}_{0}\sin 30°\,\hat{a}_y - \sin 90°\,\hat{a}_z = -\hat{a}_z$$

So, $\hat{\rho}_a = \hat{a}_\theta 1 \angle \pi/2 - kr = -\hat{a}_z 1 \angle \pi/2 - kr$

$$\hat{\rho}_w \cdot \hat{\rho}_a = \left(\hat{a}_x 0.8\angle 20° + \hat{a}_z 0.6\angle -40°\right) \cdot -\hat{a}_z 1\angle \pi/2 - kr$$

$$= -0.6(1)\angle \pi/2 - kr - 40° = -0.6\, e^{j\pi/2} e^{-jkr} e^{j40°}$$

Per (2-71), $PLF = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = (\hat{\rho}_w \cdot \hat{\rho}_a)(\hat{\rho}_w \cdot \hat{\rho}_a)^*$

$$= \left(-0.6\, e^{j\pi/2} e^{-jkr} e^{j40°}\right)\left(-0.6\, e^{-j\pi/2} e^{+jkr} e^{-j40°}\right)$$

$$PLF = 0.36 = 10\log_{10} 0.36 = -4.437\,\text{dB}$$