2.87 Repeat Problem 2.86 for Problem 2.62.

2.86 For Problem 2.61 compute the

(a) maximum effective area (in λ^2) using the computer program **Directivity** of this chapter. Compare with that computed using the equation in Table 12.1.

(b) aperture efficiencies of part (a). Are they smaller or larger than unity and why?

2.62 Repeat Problem 2.61 when the aperture distribution is that of the dominant TE_{10} mode of a rectangular waveguide, or from Table 12.1

$$E \approx \hat{\mathbf{a}}_\theta E_\theta + \hat{\mathbf{a}}_\phi E_\phi$$

$$\left.\begin{aligned} E_\theta &= -\frac{\pi}{2} C \sin\phi \frac{\cos X}{(X)^2 - \left(\frac{\pi}{2}\right)^2} \frac{\sin Y}{Y} \\ E_\phi &= -\frac{\pi}{2} C \cos\theta \cos\phi \frac{\cos X}{(X)^2 - \left(\frac{\pi}{2}\right)^2} \frac{\sin Y}{Y} \end{aligned}\right\} \begin{aligned} X &= \frac{ka}{2}\sin\theta\cos\phi \\ Y &= \frac{kb}{2}\sin\theta\sin\phi \end{aligned}$$

From problem 2.61, $a = 3\lambda$, $b = 2\lambda$, $0 \le \theta \le 90°$, and $0 \le \phi \le 360°$. By definition, $k = 2\pi/\lambda$.

- Begin by calculating the normalized radiation intensity.

Per (2-12a), $U = \frac{r^2}{2\eta}|\bar{E}|^2 = \frac{r^2}{2\eta}\left[|E_\theta|^2 + |E_\phi|^2\right] = \frac{1}{2\eta}\left[|E_\theta^\circ|^2 + |E_\phi^\circ|^2\right]$

where $\bar{E} = \bar{E}^\circ e^{-jkr}/r$

$$U = \frac{1}{2\eta}\left[\frac{\pi^2}{4} C^2 \sin^2\phi \frac{\cos^2 X}{[X^2 - (\pi/2)^2]^2} \frac{\sin^2 Y}{Y^2} + \frac{\pi^2}{4} C^2 \cos^2\theta \cos^2\phi \frac{\cos^2 X}{[X^2 - (\pi/2)^2]^2} \frac{\sin^2 Y}{Y^2}\right]$$

$$U_{norm} = \frac{U}{\frac{1}{2\eta}\frac{\pi^2}{4}C^2} = \left(\sin^2\phi + \cos^2\theta\cos^2\phi\right)\frac{\cos^2 X}{[X^2 - (\pi/2)^2]^2}\frac{\sin^2 Y}{Y^2}$$

where $X = \frac{\overset{2\pi/\lambda}{\cancel{k}}\,\overset{3\lambda}{\cancel{a}}}{2}\sin\theta\cos\phi = 3\pi\sin\theta\cos\phi \qquad 0 \le \theta \le 90°$

$Y = \frac{\overset{2\pi/\lambda}{\cancel{k}}\,\overset{2\lambda}{\cancel{b}}}{2}\sin\theta\sin\phi = 2\pi\sin\theta\sin\phi \qquad 0 \le \phi < 360°$

In order to use the Matlab program DIRECTIVITY, we need to define U.m and recognize limitations of numerical code, i.e., $\frac{\sin(Y)}{Y}$ will have a divide by zero problem when $Y=0$. Therefore, we will use the Matlab sinc() function to replace $\frac{\sin Y}{Y}$ w/ sinc(Y/pi) where $\text{sinc}(t) = \begin{cases} 1 & t=0 \\ \frac{\sin \pi t}{\pi t} & t \neq 0 \end{cases}$

The file U.m and the Matlab command window results of running DIRECTIVITY.m are shown below.

a)

```
% m-file U.m
function y = U(THETA, PHI)
X = 3*pi*sin(THETA).*cos(PHI);
Y = 2*pi*sin(THETA).*sin(PHI);
y = cos(X).^2./(X.^2-pi^2/4).^2.*sinc(Y/pi).^2.*...
(sin(PHI).^2+cos(THETA).^2.*cos(PHI).^2);
```

<u>From Command Window of MATLAB</u>

>> DIRECTIVITY

!!!WARNING: Make sure you define radiation intensity in file U.m !!!

Output device option
  Option (1): Screen
  Option (2): File

Output device=1

The lower bound of theta in degrees = 0
The upper bound of theta in degrees = 90
The lower bound of phi in degrees = 0
The upper bound of phi in degrees = 360

Input parameters:

The lower bound of theta in degrees =   0
The upper bound of theta in degrees = 90
The lower bound of phi in degrees =   0
The upper bound of phi in degrees = 360

Output parameters:

Radiated power (watts) = 0.03300
Directivity (dimensionless) = **62.53960**
Directivity (dB)   = 17.96155

a) With $D_0 = 62.5396$ from Directivity.m, we can use (2-110) $A_{em} = \frac{\lambda^2}{4\pi} D_0 = \frac{\lambda^2}{4\pi}(62.5396)$

$$\underline{\underline{A_{em} = 4.9767\,\lambda^2}} \quad \text{(DIRECTIVITY.m)}$$

From Table 12.1, $D_0 = \frac{8}{\pi^2}\left[4\pi\left(\frac{ab}{\lambda^2}\right)\right]$

$$= \frac{32}{\pi}\left[\frac{3\lambda(2\lambda)}{\lambda^2}\right] = 61.07256$$

$$A_{em} = \frac{\lambda^2}{4\pi}(61.07256) \Rightarrow \underline{\underline{A_{em} = 4.86\,\lambda^2 \quad \text{(Table 12.1)}}}$$

b) $A_p = ab = 3\lambda(2\lambda) = 6\lambda^2$

Per (2-100), $\epsilon_{ap} = \frac{A_{em}}{A_p}$

$$\epsilon_{ap} = \frac{4.9767\lambda^2}{6\lambda^2} \Rightarrow \underline{\underline{\epsilon_{ap} = 0.82946 \quad \text{(DIRECTIVITY.m)}}}$$

$$\epsilon_{ap} = \frac{4.86\lambda^2}{6\lambda^2} \Rightarrow \underline{\underline{\epsilon_{ap} = 0.81 \quad \text{(Table 12.1)}}}$$

* Both are smaller than unity. This is a consequence of the aperture <u>NOT</u> being uniformly illuminated in the TE_{10} mode, i.e., $\bar{E}_a$ field has sinusoidal distribution.