2.85 A base station is installed near your neighborhood. One of the concerns of the residents living nearby is the exposure to electromagnetic radiation. The *input power inside the transmission line feeding the base station antenna is 100 Watts* while the *omnidirectional* radiation amplitude pattern of the base station antenna can be approximated by

$$U(\theta, \phi) = B_o \sin(\theta) \qquad 0 \le \theta \le 180°, 0 \le \phi \le 360°$$

where B_o is a constant. The characteristic impedance of the transmission line feeding the base station antenna is 75 ohms while the input impedance of the base station antenna is 100 ohms. The *radiation (conduction/dielectric) efficiency of the base station antenna is 50%.* Determine the:

(a) Reflection/mismatch efficiency of the antenna (*in %*)

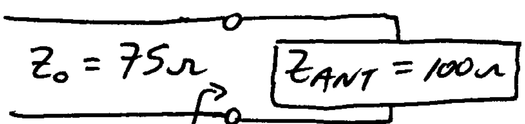
(b) Total efficiency (*in %*) of the antenna

(c) Value of B_o. *Must do the integration in closed form and show the details.*

(d) Maximum exact directivity (*dimensionless* and *in dB*)

(e) Maximum *power density* (*in Watts/cm*^{2}) at a distance of 1,000 meters. This may represent the distance from the base station to your house.

a)



$$\Gamma = \frac{Z_{ANT} - Z_o}{Z_{ANT} + Z_o} = \frac{100-75}{100+75} = 0.14286$$

From pp. 60-61 of text [eqns (2-44) + (2-45)]

$$e_r = 1 - |\Gamma|^2 = 1 - 0.14286^2 \Rightarrow \underline{e_r = 0.9796 = 97.96\%}$$

b) Per (2-44) +/or (2-45), $e_o = e_{cd}\, e_r = 0.5(0.9796) = 0.4898$

$$\hookrightarrow \underline{e_o = 48.98\%}$$

c) Per (2-13), $P_{rad} = \oiint_\Omega U \, d\Omega$ + $P_{rad} = e_o P_{in}$

$$e_o(100\,W) = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} B_o \sin\theta \sin\theta \, d\theta \, d\phi = B_o \int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi} \sin^2\theta \, d\theta$$

$$0.4898(100) = B_o(2\pi - 0)\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]\Big|_{\theta=0}^{\pi} = B_o(2\pi)\left[\left(\frac{\pi}{2} - 0\right) - (0-0)\right]$$

$$\hookrightarrow B_o = \frac{100(0.4898)}{\pi^2} \Longrightarrow \underline{B_o = 4.96267}$$

d) Per (2-16a), $D_{max} = \dfrac{4\pi U_{max}}{P_{rad}}$

From $U(\theta,\phi) = B_0 \sin\theta = 4.962675 \sin\theta \quad 0 \le \theta \le 180°$

↳ $U_{max} = 4.96267$ when $\theta = \pi/2 = 90°$

$$D_{max} = \frac{4\pi(4.96267)}{0.4898(100)} \Rightarrow \underline{\underline{D_{max} = 1.27324}}$$

$$\underline{\underline{D_{max}(dB) = 10\log_{10} 1.273 = 1.0491\ dBi}}$$

e) From (2-12), $U = r^2 W_{rad}$

↳ $W_{rad,max} = \left.\dfrac{U_{max}}{r^2}\right|_{r=1000m} = \dfrac{4.96267}{1000^2 m^2}\left(\dfrac{1 m^2}{100^2 cm^2}\right)$

$$\boxed{\begin{aligned} W_{rad,max} &= 4.96267 \times 10^{-10}\ W/cm^2 \\ &= 496.267\ pW/cm^2 \end{aligned}}$$