2.84 For an *X*-band (8.2–12.4 GHz) rectangular horn, with aperture dimensions of 5.5 cm and 7.4 cm, find its maximum effective aperture (*in* cm^2) when its gain (over isotropic) is
(a) 14.8 dB at 8.2 GHz (b) 16.5 dB at 10.3 GHz (c) 18.0 dB at 12.4 GHz

- Also, find the aperture efficiency in each case.

Per (2-111), $A_{em} = e_{cd}\left(\frac{\lambda^2}{4\pi}\right)D_0$

Note, $G_0 = e_{cd}D_0$ & $\lambda = c/f$ to get $A_{em} = G_0\frac{c^2}{4\pi f^2}$

Per (2-100), $e_{ap} = \frac{A_{em}}{A_p}$ where $A_p = 5.5(7.4) = 40.7\ cm^2$

a) $G_0 = 14.8\ dB = 10^{14.8/10}$ & $f = 8.2\times10^9\ Hz$

$$A_{em,a} = 10^{1.48}\frac{(2.9979\times10^8)^2}{4\pi(8.2\times10^9)^2} = 0.003212\ m^2$$

$$A_{em,a} = 32.122\ cm^2$$

$$e_{ap,a} = \frac{32.122}{5.5(7.4)} = 0.7892 \text{ or } 78.92\%$$

b) $G_0 = 16.5\ dB = 10^{1.65}$ & $f = 10.3\times10^9\ Hz$

$$A_{em,b} = 10^{1.65}\frac{(2.9979\times10^8)^2}{4\pi(10.3\times10^9)^2} = 0.0030113\ m^2$$

$$A_{em,b} = 30.113\ cm^2$$

$$e_{ap,b} = \frac{30.1127}{5.5(7.4)} = 0.7399 = 73.99\%$$

c) $G_0 = 18\ dB = 10^{1.8}$ & $f = 12.4\times10^9\ Hz$

$$A_{em,c} = 10^{1.8}\frac{(2.9979\times10^8)^2}{4\pi(12.4\times10^9)^2} = 0.0029348\ m^2$$

$$A_{em,c} = 29.348\ cm^2$$

$$e_{ap,c} = \frac{29.348}{5.5(7.4)} = 0.7211 = 72.11\%$$