2.76 A small circular parabolic reflector, often referred to as dish, is now being advertised as a TV antenna for direct broadcast. Assuming the diameter of the antenna is 1 meter, the frequency of operation is 3 GHz, and its aperture efficiency is 68%, determine the following:

(a) Physical area of the reflector (*in* m^2).

(b) Maximum effective area of the antenna (*in* m^2).

(c) Maximum directivity (*dimensionless* and *in dB*).

(d) Maximum power (*in watts*) that can be delivered to the TV if the power density of the wave incident upon the antenna is *10* $\mu watts/m^2$. Assume *no losses* between the incident wave and the receiver (TV).

- Hint: The physical area of the reflector means the area of the aperture not the surface area of the metal parabolic dish. Assume free space.

a) $A_p = \pi r^2 = \pi \left(\frac{1m}{2}\right)^2 \Rightarrow \underline{A_p = 0.7854\ m^2}$

b) Per (2-100), $\epsilon_{ap} = \frac{A_{em}}{A_p}$

$A_{em} = \epsilon_{ap} A_p = 0.68(0.7854) \Rightarrow \underline{A_{em} = 0.5341\ m^2}$

c) Per (2-110), $A_{em} = \frac{\lambda^2}{4\pi} D_0$ where $\lambda = c/f$

$D_0 = \frac{4\pi}{\lambda^2} A_{em} = 4\pi \left(\frac{3\times10^9}{2.9979\times10^8}\right)^2 0.5341$

$\underline{D_0 = 672.074 = 28.274\ dBi}$

d) Per (2-94), $A_e = P_T/W_i$

$P_T = P_{r,max} = A_{em} W_i = 0.5341(10\times10^{-6}\ W/m^2)$

$\underline{P_{r,max} = 5.341\times10^{-6}\ W/m^2 = 5.341\ \mu W/m^2}$