2.67 The normalized *far-field total* electric field radiated by an antenna, consisting of an infinitesimal vertical dipole (oriented along the z-axis) plus a small circular loop (parallel to the xy-plane), *placed at the origin of a spherical coordinate system* is given by:

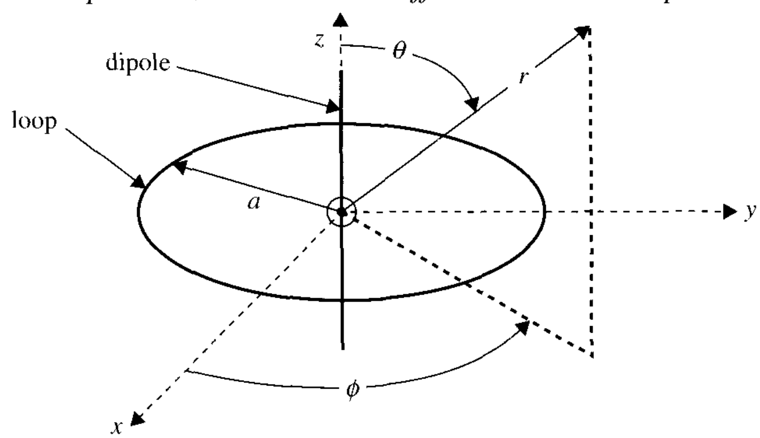
$$\overline{E}_a = (\hat{\mathbf{a}}_\theta + j2\hat{\mathbf{a}}_\phi)\sin\theta E_o \frac{e^{-jkr}}{r}; (0° \le \theta \le 180°, 0° \le \phi \le 360°)$$

Determine the:

(a) Polarization of the wave (linear, circular or elliptical). *Justify it. If circular*, state the sense of rotation. *If elliptical*, state the sense of rotation and the axial ratio (AR).

(b) partial *maximum* directivities $(D_0)_\theta$ and $(D_0)_\phi$ (*dimensionless* and *in dB*).

(c) Total maximum directivity D_o (*dimensionless* and *in dB*).
Hint: For this problem, there are two different and distinct partial directivities.



- For part a, plot/sketch polarization ellipse w/ wave propagating into page for a radiated wave traveling down the +x-axis assuming E_0 = 1000 V/m and $e^{-jkr}/r = 0.01\angle 0°$ (m^{-1}).

a) We are on +x-axis ⇒ $\theta = \pi/2$ & $\phi = 0°$.

Convert unit vectors to Cartesian coordinates:

$$\hat{a}_\theta = \cos\frac{\pi}{2}\cos 0\,\hat{a}_x + \cos\frac{\pi}{2}\sin 0\,\hat{a}_y - \sin\frac{\pi}{2}\,\hat{a}_z = -\hat{a}_z$$

$$\hat{a}_\phi = -\sin 0\,\hat{a}_x + \cos 0\,\hat{a}_y = \hat{a}_y$$

w/ these and other given values:

$$\overline{E}_a = (-\hat{a}_z + j2\,\hat{a}_y)\sin\frac{\pi}{2}(1000)(0.01\angle 0°) = j20\,\hat{a}_y - 10\,\hat{a}_z$$

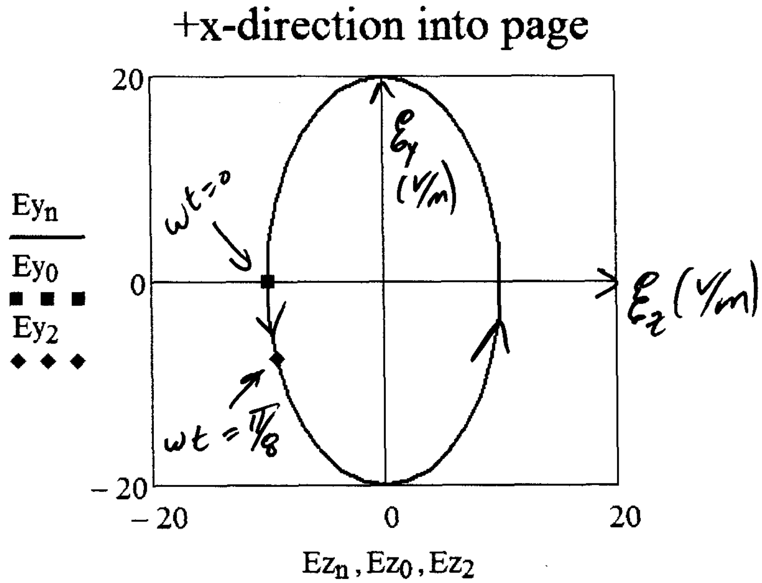
$$\overline{\mathscr{E}}_a = \mathrm{Re}\{\overline{E}_a e^{j\omega t} e^{-j\beta x}\} = 20\cos(\omega t - \beta x + \pi/2)\,\hat{a}_y - 10\cos(\omega t - \beta x)\,\hat{a}_z \ (\text{V/m})$$

The $\frac{e^{-jkr}}{r} = \frac{e^{-jkx}}{x} = 0.01\angle 0°$ term implies $e^{-jkx} = 1\angle 0°$.

So, $\underline{\overline{\mathscr{E}}_a = 20\cos(\omega t + \pi/2)\,\hat{a}_y - 10\cos(\omega t)\,\hat{a}_z \ (\text{V/m})}$

a) cont. Use MathCad to plot polarization ellipse:

$$n := 0..32 \quad wt_n := n\frac{\pi}{16} \qquad Ey_n := 20\cos\left(wt_n + \frac{\pi}{2}\right) \quad Ez_n := -10\cdot\cos(wt_n)$$



LH elliptical with an axial ratio **AR = 40/20 = 2**.

b) Per (2-12a), $U = \frac{r^2}{2\eta}|\bar{E}_a|^2 = \frac{r^2}{2\eta}\left[|E_\theta|^2 + |E_\phi|^2\right]$

$$U = \frac{r^2}{2\eta}\left[\frac{\sin^2\theta|E_0|^2}{r^2} + \frac{4\sin^2\theta|E_0|^2}{r^2}\right] = U_\theta + U_\phi$$

$$= \frac{5|E_0|^2\sin^2\theta}{2\eta} \text{ where } U_\theta = \frac{|E_0|^2\sin^2\theta}{2\eta} \text{ \& } U_\phi = \frac{4|E_0|^2\sin^2\theta}{2\eta}$$

Per (2-13), $P_{rad} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} U\sin\theta\, d\theta\, d\phi$

$$= \frac{5|E_0|^2}{2\eta}\int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi}\sin^3\theta\, d\theta$$

$$= \frac{5|E_0|^2}{2\eta}\phi\Big|_{\phi=0}^{2\pi}\left(-\cos\theta + \frac{\cos^3\theta}{3}\right)\Big|_{\theta=0}^{\pi}$$

$$= \frac{5|E_0|^2}{2\eta}(2\pi - 0)\left[(+1-1/3) - (-1+1/3)\right] = \frac{10\pi|E_0|^2}{2\eta}(4/3)$$

$$= \frac{20\pi|E_0|^2}{3\eta}$$

b) cont. Use (2-17a) & (2-17b) to get maximum partial directivities.

$$D_{\theta,max} = \frac{4\pi U_{\theta,max}}{P_{rad}} = \frac{4\pi \left(\frac{|E_0|^2 \sin^2\theta}{2\eta} \right)\Big|_{max}}{\frac{20\pi |E_0|^2}{3\eta}}$$

$$\underline{\underline{D_{\theta,max} = 0.3 = 10\log_{10} 0.3 = -5.229\text{ dBi} \ @\ \theta = 90° = \pi/2}}$$

$$D_{\phi,max} = \frac{4\pi U_{\phi,max}}{P_{rad}} = \frac{4\pi \left(\frac{4|E_0|^2 \sin^2\theta}{2\eta} \right)\Big|_{max}}{\frac{20\pi |E_0|^2}{3\eta}}$$

$$\underline{\underline{D_{\phi,max} = 1.2 = 10\log_{10} 1.2 = 0.7918\text{ dBi} \ @\ \theta = 90° = \pi/2}}$$

c) Per (2-17), the total directivity is

$$D_0 = D_{max} = D_{\theta,max} + D_{\phi,max} = 0.3 + 1.2$$

$$\underline{\underline{D_{max} = 1.5 = 10\log_{10} 1.5 = 1.7609\text{ dBi}}}$$