**2.57** The input reactance of an infinitesimal linear dipole of length $\lambda/60$ and radius $a = \lambda/200$ is given by

$$X_{in} \simeq -120\frac{[\ln(l/2a) - 1]}{\tan(kl/2)}$$

Assuming the wire of the dipole is copper with a conductivity of $5.7 \times 10^7$ S/m, determine at $f = 1$ GHz the

(a) loss resistance (b) radiation resistance (c) radiation efficiency

(d) VSWR when the antenna is connected to a 50-ohm line

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- with length changed to $\lambda/50$ and radius to $\lambda/250$ (Hint: look at Chapter 4)

a) → Since $l = \lambda/50$, we can use the constant current approx.

Per (2-90b) $R_L = R_{hf} = \frac{l}{P}\sqrt{\frac{\omega\mu_0}{2\sigma}} = \frac{\lambda/50}{2\pi(\lambda/250)}\sqrt{\frac{2\pi(1\times10^9)4\pi\times10^{-7}}{2(5.7\times10^7)}}$

$$\underline{\underline{R_L = 0.0066226\,\Omega = 6.6227\text{ m}\Omega}}$$

b) Use (4-19) to get radiation resistance

$R_r = 80\pi^2\left(\frac{l}{\lambda}\right)^2 = 80\pi^2\left(\frac{\lambda/50}{\lambda}\right)^2 \Rightarrow \underline{\underline{R_r = 0.31583\,\Omega}}$

c) Per (2-90), the radiation efficiency is

$e_{cd} = \frac{R_r}{R_r + R_L} = \frac{0.31583}{0.31583 + 0.0066} = \underline{\underline{0.97946 = 97.95\%}}$

d) $X_{in} \simeq -120\,\frac{\ln\left(\frac{\lambda/50}{2\,\lambda/250}\right) - 1}{\tan\left(\frac{2\pi}{\lambda}\,\lambda/50/2\right)} = 159.6624859\,\Omega$

$Z_{ANT} = R_L + R_r + jX_{in} = 0.31583 + 0.0066227 + j159.66249\,\Omega$

$= 0.32245 + j159.6625\,\Omega$

$\Gamma_{ANT} = \frac{Z_{ANT} - Z_0}{Z_{ANT} + Z_0} = \frac{(0.32245 + j159.66) - 50}{(0.32245 + j159.66) + 50}$

$= 0.998848735\angle 34.777^\circ$

$VSWR = \frac{1 + |\Gamma_{ANT}|}{1 - |\Gamma_{ANT}|} = \frac{1 + 0.99885}{1 - 0.99885} \Rightarrow \underline{\underline{VSWR = 1736.22}}$

(essentially $\infty$)