2.57 The input reactance of an infinitesimal linear dipole of length $\lambda/60$ and radius $a = \lambda/200$ is given by

$$X_{in} \simeq -120\frac{[\ln(l/2a) - 1]}{\tan(kl/2)}$$

Assuming the wire of the dipole is copper with a conductivity of 5.7×10^7 S/m, determine at $f = 1$ GHz the (a) loss resistance (b) radiation resistance (c) radiation efficiency (d) VSWR when the antenna is connected to a 50-ohm line

- Change the wire conductivity to that of brass $\sigma_{brass} = 1.1\times10^7$ S/m. Hint: Look at Chapter 4 section on infinitesimal dipoles.

a) Since $\ell = \lambda/60 < \lambda/50$, we can use the constant current approximation, per (2-90b)

$$R_L = R_{hf} = \frac{\ell}{P}\sqrt{\frac{\omega\mu_0}{2\sigma}} = \frac{\lambda/60}{2\pi\,\lambda/200}\sqrt{\frac{2\pi\,10^9(4\pi\times10^{-7})}{2(1.1\times10^7)}}$$

$$\underline{\underline{R_L = 0.01005\,\Omega = 10.0504\,m\Omega}}$$

b) Use (4-19) for the radiation resistance

$$R_r = 80\pi^2\left(\frac{\ell}{\lambda}\right)^2 = 80\pi^2\left(\frac{1}{60}\right)^2 \Rightarrow \underline{\underline{R_r = 0.21932\,\Omega}}$$

c) Per (2-90), the radiation efficiency is

$$e_{cd} = \frac{R_r}{R_r + R_L} = \frac{0.21932}{0.21932 + 0.01005} \Rightarrow \underline{\underline{e_{cd} = 0.95618 = 95.6\%}}$$

d) $$X_{in} \simeq -120\frac{\ln\left(\frac{\lambda/60}{2(\lambda/200)}\right) - 1}{\tan\left(\frac{2\pi}{\lambda}\frac{\lambda}{60}\frac{1}{2}\right)} = 1120.08038\,\Omega$$

$$Z_{ant} = R_r + R_L + jX_{in} = 0.229375 + j1120.08038\,\Omega$$

$$|\Gamma_{ant}| = \left|\frac{Z_{ant} - 50}{Z_{ant} + 50}\right| = \left|\frac{0.229375 + j1120.08 - 50}{0.229375 + j1120.08 + 50}\right| = 0.99998175 4$$

$$VSWR = \frac{1 + |\Gamma_{ant}|}{1 - |\Gamma_{ant}|} = \frac{1.999982}{1 - 0.999982} \Rightarrow \underline{\underline{VSWR = 109{,}609.2}}$$

(essentially ∞)