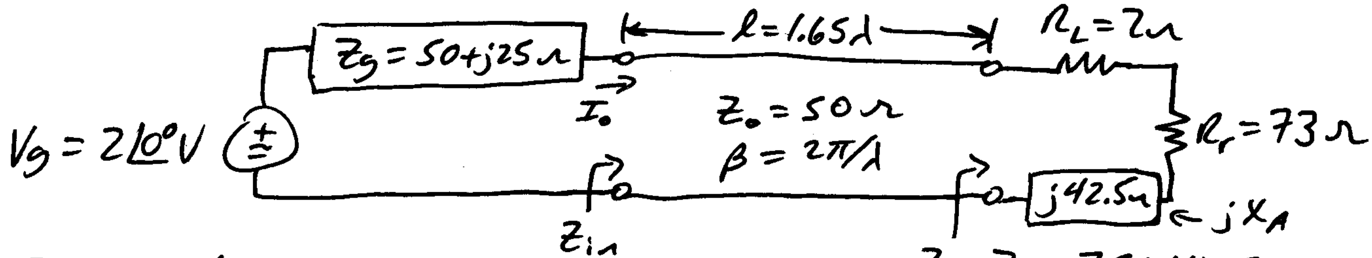
2.53 A $\lambda/2$ dipole, with a total loss resistance of 2 ohm, is connected to a generator whose internal impedance is $50 + j25$ ohms. Assuming that the peak voltage of the generator is 2 V and the impedance of the dipole, excluding the loss resistance, is $73 + j42.5$ ohms, find the power

(a) supplied by the source (real)
(b) radiated by the antenna
(c) dissipated by the antenna

- Assume loss resistance is 2 Ω and the generator has a lossless 50 Ω transmission line of length 1.65λ connecting it to the antenna.

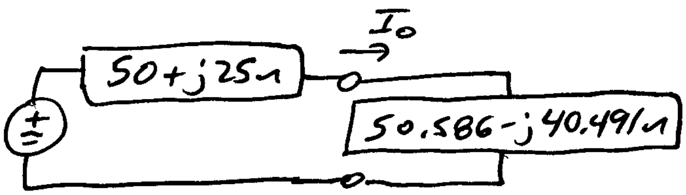


From notes - $Z_L = Z_A = 75 + j42.5\,\Omega$

$$Z_{in} = Z_o\left[\frac{Z_L + jZ_o\tan(\beta\ell)}{Z_o + jZ_L\tan(\beta\ell)}\right] = 50\left[\frac{(75+j42.5) + j50\tan(\frac{2\pi}{\lambda}1.65\lambda)}{50 + j(75+42.5)\tan(3.3\pi)}\right]$$

$$Z_{in} = 50.58613 - j40.49062\,\Omega$$

Equivalent Circuit



$$I_o = \frac{2\angle 0°}{(50+j25) + (50.586 - j40.491)} = 0.0196518\angle 8.755°\ A$$

a) $P_{source} = \frac{1}{2}\mathrm{Re}\{V_g I_o^*\} = \frac{1}{2}\mathrm{Re}\{2\angle 0°(0.01965\angle -8.755°)\}$

$$\underline{\underline{P_{source} = 19.4228\ mW}}$$

For a lossless TL, $P_{in} = P_{ant}$, $P_{loss} = \frac{R_L}{R_r + R_L}P_{ant}$, & $P_r = \frac{R_r}{R_r + R_L}P_{ant}$

$$P_{ant} = P_{in} = \frac{1}{2}|I_o|^2 R_{in} = \frac{1}{2}(0.01965)^2\,50.586 = 9.768\ mW$$

b) $P_{rad} = \frac{73}{73+2}(9.768) \Rightarrow \underline{\underline{P_{rad} = 9.5075\ mW}}$

c) $P_{loss} = \frac{2}{73+2}(9.768) \Rightarrow \underline{\underline{P_{loss} = 0.2605\ mW}}$