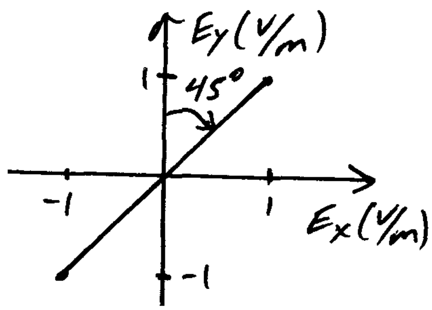
2.44 A linearly polarized wave traveling in the negative z-direction has a tilt angle (τ) of 45°. It is incident upon an antenna whose polarization characteristics are given by

$$\hat{\boldsymbol{\rho}}_a = \frac{4\hat{\mathbf{a}}_x + j\hat{\mathbf{a}}_y}{\sqrt{17}}$$

Find the polarization loss factor PLF (*dimensionless* and *in dB*).

- In addition, sketch the polarization ellipse of the wave for wave propagating into page assuming $E_x = E_y = 1$ V/m and find $\hat{\rho}_w$.

For wave propagating into the page, choose $+E_y$ for vertical axis and $+E_x$ for horizontal axis.



From polarization ellipse, we expect the phasor vector electric field in the -z-direction:

$$\bar{E}_i = \hat{\rho}_w E_i = (E_x \hat{a}_x + E_y \hat{a}_y)\, e^{+jkz} = (\hat{a}_x + \hat{a}_y)\, e^{+jkz} \text{ V/m}$$

$$E_i^2 = \bar{E}_i \cdot \bar{E}_i^* = (\hat{a}_x + \hat{a}_y) e^{jkz} \cdot (\hat{a}_x + \hat{a}_y) e^{-jkz} = (1+0+0+1) e^0 = 2$$

$$\bar{\rho}_w = \frac{\bar{E}_i}{E_i} = \frac{(\hat{a}_x + \hat{a}_y) e^{+jkz}}{\sqrt{2}} \Rightarrow \underline{\underline{\bar{\rho}_w = \frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}} e^{jkz}}}$$

$$(2\text{-}71)\quad PLF = |\bar{\rho}_w \cdot \hat{\rho}_a|^2 = (\hat{\rho}_w \cdot \hat{\rho}_a)(\hat{\rho}_w \cdot \hat{\rho}_a)^*$$

$$= \left(\frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}} e^{jkz} \cdot \frac{4\hat{a}_x + j\hat{a}_y}{\sqrt{17}}\right)\left(\frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}} e^{-jkz} \cdot \frac{4\hat{a}_x - j\hat{a}_y}{\sqrt{17}}\right)$$

$$= \left(\frac{(4+j)}{\sqrt{34}} e^{jkz}\right)\left(\frac{4-j}{\sqrt{34}} e^{-jkz}\right) = \frac{16+1}{34} e^0$$

$$\underline{\underline{PLF = 0.5 = -3.0103 \text{ dB}}}$$