2.37 A 300 MHz uniform plane wave, traveling along the *x-axis in the negative x-direction*, whose electric field is given by

$$\mathbf{E}_w = E_o(j\hat{\mathbf{a}}_y + 3\hat{\mathbf{a}}_z)e^{+jkx}$$

where E_0 is a real constant, impinges upon a dipole antenna that is placed at the origin and whose electric field radiated toward the *x-axis in the positive x-direction* is given by

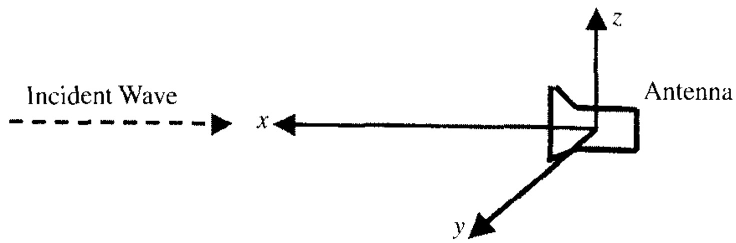
$$\mathbf{E}_a = E_a(\hat{\mathbf{a}}_y + 2\hat{\mathbf{a}}_z)e^{-jkx}$$

where E_a is a real constant. Determine the following:

(a) Polarization of the incident wave (*including axial ratio and sense of rotation, if any*). You must justify (state why?).

(b) Polarization of the antenna (*including axial ratio and sense of rotation, if any*). You must justify (state why?).

(c) Polarization loss factor (*dimensionless* and *in dB*).



- For plots, $E_0 = E_a = 1$ V/m. For part (a), write-out a time-domain eq'n for the electric field, plot polarization ellipse w/ wave propagating out of page, annotate RH/LH, and find tilt angle with respect to the $+E_y$-axis. For part (b), write-out a time-domain eq'n for the electric field, plot polarization ellipse w/ wave propagating into page, annotate RH/LH, and find tilt angle with respect to the $+E_y$-axis. For part (c), also find $\hat{\rho}_w$ & $\hat{\rho}_a$.

a) $\bar{\mathcal{E}}_w = \mathcal{Re}\{\bar{E}_w e^{j\omega t}\} = \mathcal{Re}\{E_o(j\hat{a}_y + 3\hat{a}_z)e^{+jkx}e^{j\omega t}\}$

$= E_o\,\mathcal{Re}\{\hat{a}_y e^{j\pi/2}e^{jkx}e^{j\omega t} + \hat{a}_z 3 e^{jkx}e^{j\omega t}\}$

$$\underline{\underline{\bar{\mathcal{E}}_w = \hat{a}_y E_o\cos(\omega t + kx + \pi/2) + \hat{a}_z 3E_o\cos(\omega t + kx)}}$$

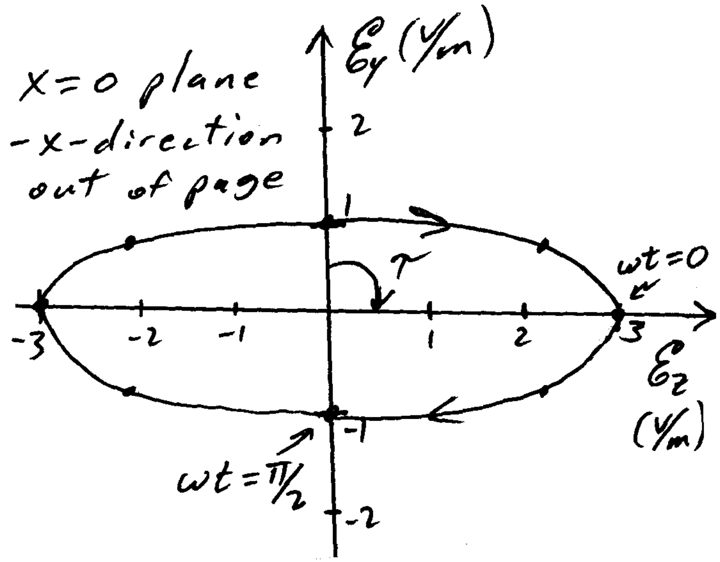
Since $E_y = E_o \neq E_z = 3E_o$ and $\phi_y - \phi_z = \pi/2$, expect the wave to be elliptically polarized.

For simplicity, choose $x=0$ plane & $E_o = 1\frac{V}{m}$ for plot.

$\bar{\mathcal{E}}_w(x=0,t) = \hat{a}_y\cos(\omega t + \pi/2) + \hat{a}_z 3\cos(\omega t)$ (V/m)

For $-x$-direction <u>out of</u> page, choose $\mathcal{E}_y$ vertical and $\mathcal{E}_z$ to right as $\hat{a}_z \times \hat{a}_y = -\hat{a}_x$.

ωt	$\mathcal{E}_y = \cos(\omega t + \pi/2)$ (V/m)	$\mathcal{E}_z = 3\cos(\omega t)$ (V/m)
0	0	3
$\pi/4$	-0.707	2.12
$\pi/2$	-1	0
$\frac{3\pi}{4}$	-0.707	-2.12
π	0	-3
$5\pi/4$	0.707	-2.12
$3\pi/2$	1	0
$\frac{7\pi}{4}$	0.707	2.12



LH elliptical, $\tau = 90°$, & $AR = \frac{3+3}{1+1} \Rightarrow AR = 3$

b) $\bar{\mathcal{E}}_a = \mathcal{Re}\{\bar{E}_a e^{j\omega t}\} = \mathcal{Re}\{E_a(\hat{a}_y + 2\hat{a}_z)e^{-jkx}e^{j\omega t}\}$

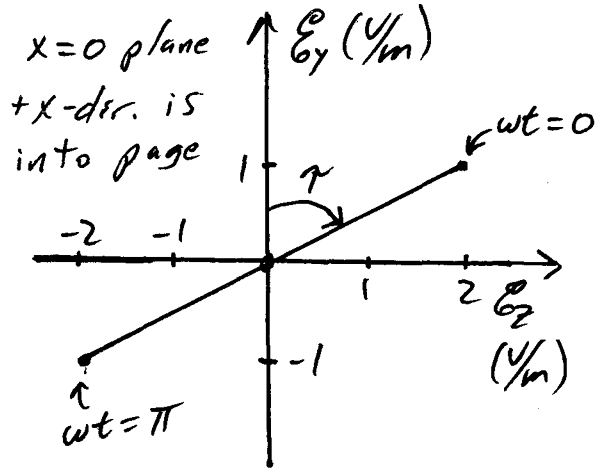
$$\bar{\mathcal{E}}_a = \hat{a}_y E_a \cos(\omega t - kx) + \hat{a}_z 2 E_a \cos(\omega t - kx)$$

Since $\mathcal{E}_y$ & $\mathcal{E}_z$ are in-phase ($\phi_y - \phi_z = 0$), expect the antenna to be linearly polarized. Choose $x=0$ plane for plot w/ $E_a = 1$ V/m:

$$\bar{\mathcal{E}}_a(x=0,t) = \hat{a}_y \cos(\omega t) + \hat{a}_z 2\cos(\omega t) \text{ (V/m)}$$

For +x-direction into page, choose $\mathcal{E}_y$ vertical and $\mathcal{E}_z$ to the right as $\hat{a}_y \times \hat{a}_z = +\hat{a}_x$.

ωt	$\mathcal{E}_y = \cos(\omega t)$ V/m	$\mathcal{E}_z = 2\cos(\omega t)$ V/m
0	1	2
$\pi/4$	0.707	1.414
$\pi/2$	0	0
$3\pi/4$	-0.707	-1.414
π	-1	-2



Linearly polarized, sense N/A, $AR \to \infty$

tilt angle $\tau = \tan^{-1}(2/1) = 63.435°$

c) $|\bar{E}_w|^2 = \bar{E}_w \cdot \bar{E}_w^* = E_0(j\hat{a}_y + 3\hat{a}_z)e^{jkx} \cdot E_0(-j\hat{a}_y + 3\hat{a}_z)e^{-jkx}$

$|\bar{E}_w| = \sqrt{E_0^2(+1+9)} = \sqrt{10}\,E_0$

$$\hat{\rho}_w = \frac{\bar{E}_w}{|\bar{E}_w|} = \frac{E_0(j\hat{a}_y + 3\hat{a}_z)e^{jkx}}{\sqrt{10}\,E_0} \Rightarrow \hat{\rho}_w = \left(\frac{j\hat{a}_y + 3\hat{a}_z}{\sqrt{10}}\right)e^{jkx}$$

$|\bar{E}_a|^2 = \bar{E}_a \cdot \bar{E}_a^* = E_a(\hat{a}_y + 2\hat{a}_z)e^{-jkx} \cdot E_a(\hat{a}_y + 2\hat{a}_z)e^{+jkx}$

$|\bar{E}_a| = \sqrt{E_a^2(1^2+2^2)} = \sqrt{5}\,E_a$

$$\hat{\rho}_a = \frac{\bar{E}_a}{|\bar{E}_a|} = \frac{E_a(\hat{a}_y + 2\hat{a}_z)e^{-jkx}}{\sqrt{5}\,E_a} \Rightarrow \hat{\rho}_a = \left(\frac{\hat{a}_y + 2\hat{a}_z}{\sqrt{5}}\right)e^{-jkx}$$

$$(2\text{-}71)\quad PLF = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left|\left(\frac{j\hat{a}_y + 3\hat{a}_z}{\sqrt{10}}\right)e^{jkx} \cdot \left(\frac{\hat{a}_y + 2\hat{a}_z}{\sqrt{5}}\right)e^{-jkx}\right|^2$$

$$= \left|\frac{j + 0 + 0 + 3(2)}{\sqrt{10}\sqrt{5}}\,e^0\right|^2 = \frac{6+j}{\sqrt{50}} \cdot \frac{6-j}{\sqrt{50}}$$

$$= \frac{36+1}{50} = \frac{37}{50}$$

$$PLF = 0.74 = -1.3077\text{ dB}$$