**2.33** A uniform plane wave, of a form similar to (2-55), is traveling in the positive $z$-direction. Find the polarization (linear, circular, or elliptical), sense of rotation (CW or CCW), axial ratio (AR), and tilt angle $\tau$ (in degrees) when
(b) $E_x \neq E_y$, $\Delta\phi = \phi_y - \phi_x = 0$
In all cases, justify the answer.

- Assume $E_x = 0.5$ V/m and $E_y = 1$ V/m.
- Also, write-out a time-domain equation for the electric field, plot the polarization ellipse, annotate RH/LH instead of CW/CCW, and find tilt angle wrt the $+\mathcal{E}_y$ - axis.

$$\bar{\mathcal{E}}(z,t) = \hat{a}_x E_x \cos(\omega t - kz + \phi_x) + \hat{a}_y E_y \cos(\omega t - kz + \phi_y)$$

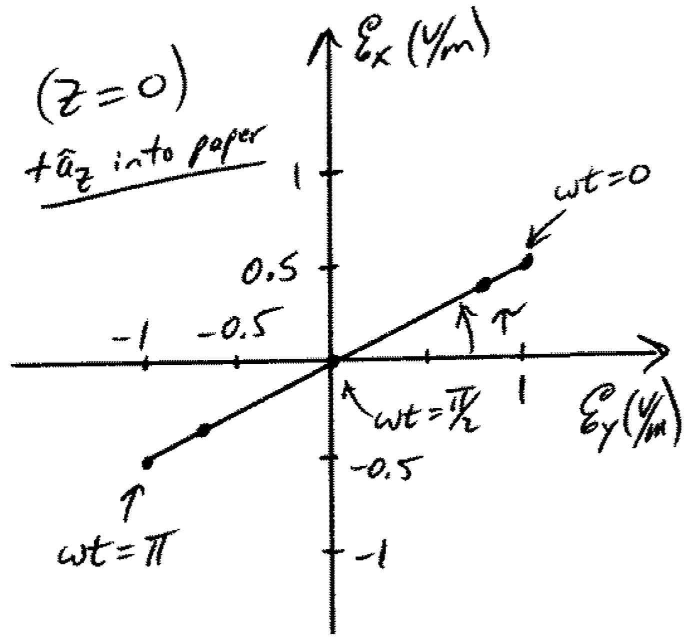
+z-dir.

For simplicity, let $\phi_y = \phi_x = 0°$.

$$\underline{\underline{\bar{\mathcal{E}}(z,t) = \hat{a}_x\, 0.5 \cos(\omega t - kz) + \hat{a}_y \cos(\omega t - kz) \ \text{V/m}}}$$

Evaluate $\bar{\mathcal{E}}$ at $z=0$

| $\omega t$ | $\bar{\mathcal{E}} = \hat{a}_x 0.5\cos(\omega t) + \hat{a}_y \cos(\omega t)$ |
|---|---|
| $0$ | $0.5\hat{a}_x + \hat{a}_y$ |
| $\pi/4$ | $0.3536\hat{a}_x + 0.707\hat{a}_y$ |
| $\pi/2$ | $0$ |
| $3\pi/4$ | $-0.3536\hat{a}_x - 0.707\hat{a}_y$ |
| $\pi$ | $-0.5\hat{a}_x - \hat{a}_y$ |




<u>Linear Polarization</u>  <u>Sense N/A</u>

<u>AR = $\infty$</u>  <u>Tilt angle $\tau = \tan^{-1}\left(\frac{0.5}{1}\right) = 26.565°$</u>

**2.33** A uniform plane wave, of a form similar to (2-55), is traveling in the positive $z$-direction. Find the polarization (linear, circular, or elliptical), sense of rotation (CW or CCW), axial ratio (AR), and tilt angle $\tau$ (in degrees) when

(c) $E_x = E_y, \Delta\phi = \phi_y - \phi_x = \pi/2$

In all cases, justify the answer.

- Assume $E_x = E_y = 1$ V/m.
- Also, write-out a time-domain equation for the electric field, plot the polarization ellipse, annotate RH/LH instead of CW/CCW, and find tilt angle wrt the $+\mathcal{E}_y$ - axis.

$$\bar{\mathcal{E}} = \hat{a}_x E_x \cos(\omega t - kz + \phi_x) + \hat{a}_y E_y \cos(\omega t - kz + \phi_y)$$

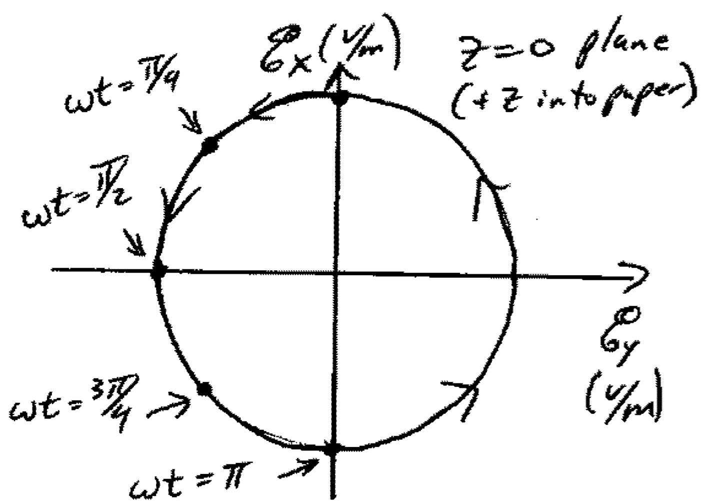
let $E_x = E_y = 1 \frac{V}{m}, \ \phi_x = 0, \ \phi_y = \pi/2$

$$\underline{\underline{\bar{\mathcal{E}} = \hat{a}_x \cos(\omega t - kz) + \hat{a}_y \cos(\omega t - kz + \tfrac{\pi}{2}) \tfrac{V}{m}}}$$

Evaluate $\bar{\mathcal{E}}$ vs $\omega t$ @ $z=0$

$$\Rightarrow \underline{\bar{\mathcal{E}} = \hat{a}_x \cos(\omega t) + \hat{a}_y \cos(\omega t + \pi/2) \tfrac{V}{m}}$$

| $\omega t$ | $\bar{\mathcal{E}}$ |
|---|---|
| 0 | $\hat{a}_x$ |
| $\pi/4$ | $0.707\hat{a}_x - 0.707\hat{a}_y$ |
| $\pi/2$ | $-\hat{a}_y$ |
| $3\pi/4$ | $-0.707\hat{a}_x - 0.707\hat{a}_y$ |
| $\pi$ | $-\hat{a}_x$ |
| ⋮ | |




<u>LH Circular Polarization</u>

$$\underline{\underline{AR = \frac{1}{1} = 1}}$$

(2-68) $\tau = \pi/2 - \frac{1}{2}\tan^{-1}\left[\frac{2E_xE_y}{E_x^2 - E_y^2}\cos(\Delta\phi)\right] = \underline{\pi/2}$

↪ Really is <u>undefined</u> for <u>Circular</u> polarization

**2.33** A uniform plane wave, of a form similar to (2-55), is traveling in the positive $z$-direction. Find the polarization (linear, circular, or elliptical), sense of rotation (CW or CCW), axial ratio (AR), and tilt angle $\tau$ (in degrees) when

(f) $E_x = E_y, \Delta\phi = \phi_y - \phi_x = -\pi/4$

In all cases, justify the answer.

- Assume $E_x = E_y = 1$ V/m.
- Also, write-out a time-domain equation for the electric field, plot the polarization ellipse, annotate RH/LH instead of CW/CCW, and find tilt angle wrt the $+\mathcal{E}_y$ - axis.

$$\bar{\mathcal{E}} = \hat{a}_x E_x \cos(\omega t - kz + \phi_x) + \hat{a}_y E_y \cos(\omega t - kz + \phi_y)$$

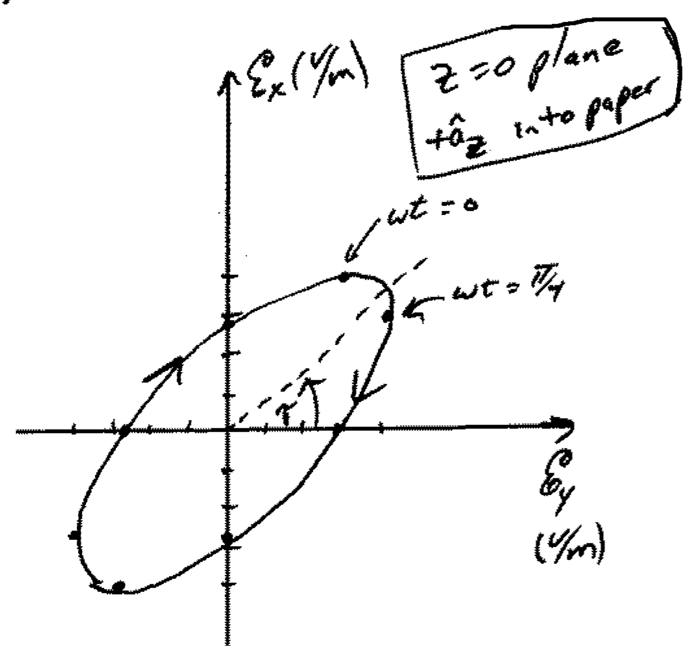
let $E_x = E_y = 1 \frac{V}{m}$, $\phi_x = 0$, $\phi_y = -\pi/4$

$$\underline{\bar{\mathcal{E}}(z,t) = \hat{a}_x \cos(\omega t - kz) + \hat{a}_y \cos(\omega t - kz - \pi/4) \ (V/m)}$$

choose $z = 0$ plane

$$\bar{\mathcal{E}}(0,t) = \hat{a}_x \cos(\omega t) + \hat{a}_y \cos(\omega t - \pi/4) \ (V/m)$$

| $\omega t$ (rad) | $\mathcal{E}_x$ (V/m) | $\mathcal{E}_y$ (V/m) |
|---|---|---|
| 0 | 1 | 0.707 |
| $\pi/4$ | 0.707 | 1 |
| $\pi/2$ | 0 | 0.707 |
| $3\pi/4$ | -0.707 | 0 |
| $\pi$ | -1 | -0.707 |
| $5\pi/4$ | -0.707 | -1 |
| $3\pi/2$ | 0 | -0.707 |
| $7\pi/4$ | 0.707 | 0 |




**RH Elliptical Polarization**

$$AR = \frac{\text{major axis}}{\text{minor axis}} = \frac{1.30656}{0.541196} = \underline{2.414} \quad \left(\text{use } 2\text{-}66 \ \& \ 2\text{-}67\right)$$

tilt angle $= \underline{\tau = 45°}$ (by inspection or 2-68)