2.33 A uniform plane wave, of a form similar to (2-55), is traveling in the positive z-direction. Find the polarization (linear, circular, or elliptical), sense of rotation (CW or CCW), axial ratio (AR), and tilt angle τ (in degrees) when

(a) $E_x = E_y, \Delta\phi = \phi_y - \phi_x = 0$ In all cases, justify the answer.

- Assume $E_x = E_y = 1$ V/m.
- Also, write-out a time-domain equation for the electric field, plot the polarization ellipse w/ wave propagating into page, annotate RH/LH instead of CW/CCW, and find tilt angle with respect to the $+E_y$ -axis.

$$\bar{\mathcal{E}} = \hat{a}_x E_x \cos(\omega t - kz + \phi_x) + \hat{a}_y E_y \cos(\omega t - kz + \phi_y)$$

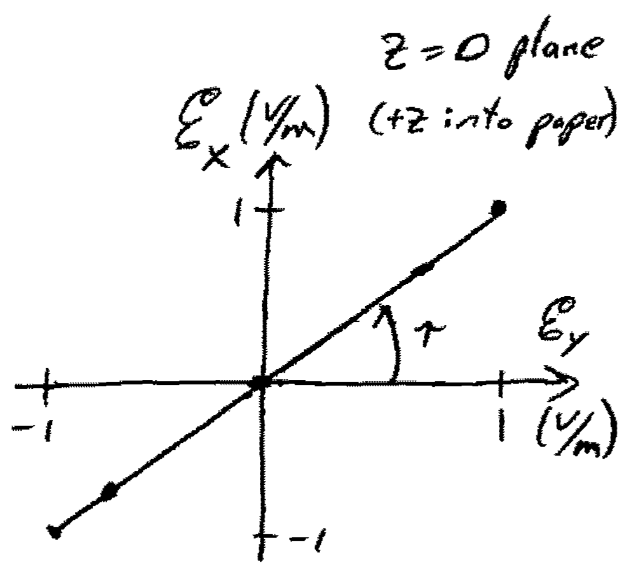
let $E_x = E_y = 1$ V/m and $\phi_x = \phi_y = 0$

$$\bar{\mathcal{E}} = \hat{a}_x \cos(\omega t - kz) + \hat{a}_y \cos(\omega t - kz) \ \frac{V}{m}$$

Evaluate $\bar{\mathcal{E}}$ @ $z = 0$ vs ωt

ωt	$\bar{\mathcal{E}} = \hat{a}_x \cos(\omega t) + \hat{a}_y \cos(\omega t)$ (V/m)
0	$\hat{a}_x + \hat{a}_y$
$\pi/4$	$0.707\,\hat{a}_x + 0.707\,\hat{a}_y$
$\pi/2$	0
$3\pi/4$	$-0.707\,\hat{a}_x - 0.707\,\hat{a}_y$
π	$-\hat{a}_x - \hat{a}_y$

plot →



Linear Polarization

Sense N/A

AR = ∞

Tilt angle $\tau = 45°$ (By inspection)

2.33 A uniform plane wave, of a form similar to (2-55), is traveling in the positive z-direction. Find the polarization (linear, circular, or elliptical), sense of rotation (CW or CCW), axial ratio (AR), and tilt angle τ (in degrees) when

(d) $E_x = E_y$, $\Delta\phi = \phi_y - \phi_x = -\pi/2$ In all cases, justify the answer.

- Assume $E_x = E_y = 1$ V/m.
- Also, write-out a time-domain equation for the electric field, plot the polarization ellipse w/ wave propagating into page, annotate RH/LH instead of CW/CCW, and find tilt angle with respect to the $+E_y$-axis.

$$\bar{\mathscr{E}}(z,t) = \hat{a}_x E_{x_0}\cos(\omega t - kz + \phi_x) + \hat{a}_y E_{y_0}\cos(\omega t - kz + \phi_y)$$

(propagates in $+\hat{a}_z$ direction)

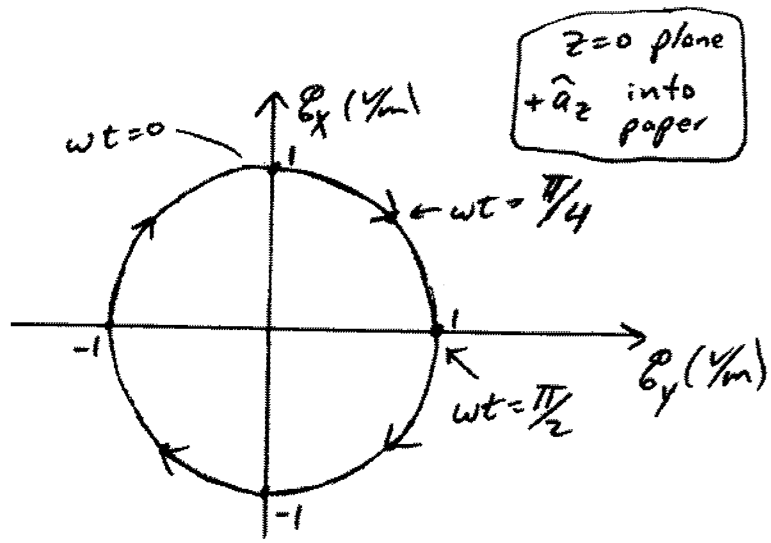
letting $E_x = E_y = 1$ V/m, $\phi_x = 0$, & $\phi_y = -\pi/2$

$$\bar{\mathscr{E}}(z,t) = \hat{a}_x \cos(\omega t - kz) + \hat{a}_y \cos(\omega t - kz - \pi/2) \text{ V/m}$$

Evaluate $\bar{\mathscr{E}}$ @ $z = 0$ for $0 \le \omega t < 2\pi$

$$\bar{\mathscr{E}}(0,t) = \hat{a}_x \cos(\omega t) + \hat{a}_y \cos(\omega t - \pi/2) \text{ V/m}$$

ωt	$\mathscr{E}_x$ (V/m)	$\mathscr{E}_y$ (V/m)
0	1	0
$\pi/4$	0.707	0.707
$\pi/2$	0	1
$3\pi/4$	-0.707	0.707
π	-1	0
$5\pi/4$	-0.707	-0.707
$3\pi/2$	0	-1
$7\pi/4$	0.707	-0.707



Circular Polarization

RH sense **AR = 1**

Tilt angle τ N/A for circle

2.33 A uniform plane wave, of a form similar to (2-55), is traveling in the positive z-direction. Find the polarization (linear, circular, or elliptical), sense of rotation (CW or CCW), axial ratio (AR), and tilt angle τ (in degrees) when

(h) $E_x = 0.5E_y$, $\Delta\phi = \phi_y - \phi_x = -\pi/2$ In all cases, justify the answer.

- Assume $E_y = 1$ V/m $\Rightarrow$ $E_x = 0.5E_y = 0.5$ V/m.
- Also, write-out a time-domain equation for the electric field, plot the polarization ellipse w/ wave propagating into page, annotate RH/LH instead of CW/CCW, and find tilt angle with respect to the + E_y -axis.

$$\bar{\mathcal{E}}(z,t) = \hat{a}_x E_x \cos(\omega t - kz + \phi_x) + \hat{a}_y E_y \cos(\omega t - kz + \phi_y)$$

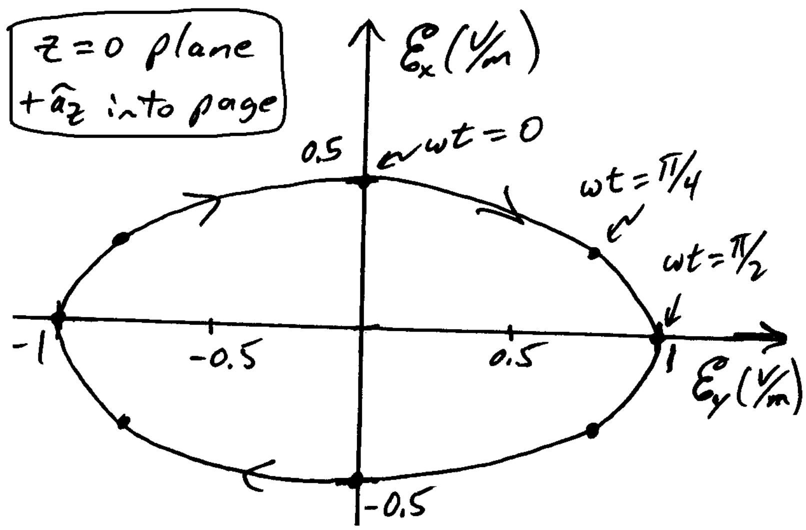
let $E_y = 1$ V/m & $E_x = 0.5$ V/m, $\phi_x = 0$, & $\phi_y = -\pi/2$

$$\bar{\mathcal{E}}(z,t) = \hat{a}_x 0.5\cos(\omega t - kz) + \hat{a}_y \cos(\omega t - kz - \pi/2) \text{ V/m}$$

Evaluate $\bar{\mathcal{E}}$ @ $z=0$ for $0 \le \omega t < 2\pi$

$$\bar{\mathcal{E}}(0,t) = \hat{a}_x 0.5\cos(\omega t) + \hat{a}_y \cos(\omega t - \pi/2) \text{ V/m}$$

ωt	$\mathcal{E}_x$ (V/m)	$\mathcal{E}_y$ (V/m)
0	0.5	0
$\pi/4$	0.354	0.707
$\pi/2$	0	1
$3\pi/4$	-0.354	0.707
π	-0.5	0
$5\pi/4$	-0.354	-0.707
$3\pi/2$	0	-1
$7\pi/4$	0.354	-0.707



Elliptical polarization

RH sense

$AR = \frac{1+1}{0.5+0.5} \Rightarrow$ **$AR = 2$**

Tilt angle **$\tau = 0$** by inspection