2.31 The normalized field pattern of the main beam of a conical horn antenna, mounted on an infinite ground plane with z perpendicular to the aperture, is given by

$$\frac{J_1(ka\sin\theta)}{\sin\theta}$$

where a is its radius at the aperture. Assuming that $a = \lambda$, find the

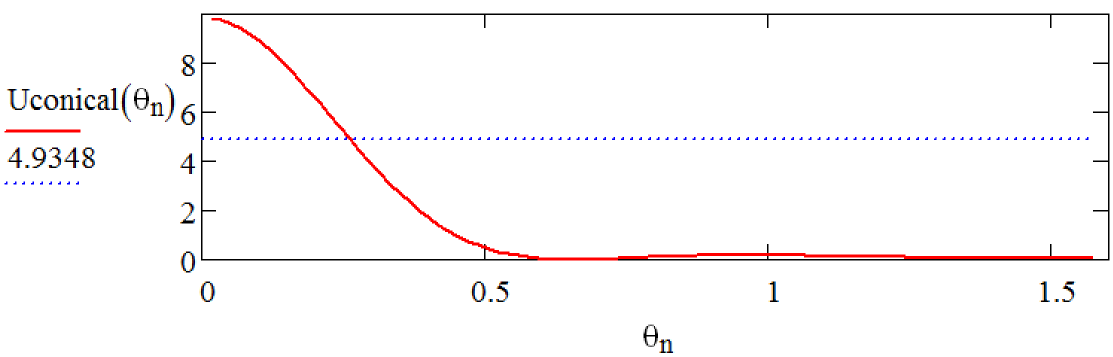
(a) half-power beamwidth

(b) directivity using Kraus' approximate formula

- Since the normalized electric field $|\boldsymbol{E}|_{norm} = J_1(ka\sin\theta)/\sin\theta$ and radiation intensity $U \propto |\boldsymbol{E}|^2$, we can use a plot of $[J_1(ka\sin\theta)/\sin\theta]^2$ to determine the HPBW. I used MathCad for the plotting and calculations.

a) When $a = \lambda$, $ka = (2\pi/\lambda)\lambda = 2\pi$. Plot $U = [J_1[ka\sin(\theta)]/\sin(\theta)]^2$ versus θ (range from 0 to $\pi/2$) in order to determine approximate maximum and half-power angles.

$n := 0..90$ $\theta_n := \frac{\pi}{180}\cdot n$ $\text{Uconical}(ang) := \left(\frac{J1(2\pi\cdot\sin(ang))}{\sin(ang)}\right)^2$



$\text{Umax} := \text{Uconical}(10^{-10})$ $\text{Umax} = 9.869604$

$\text{Uhalf} := \frac{\text{Umax}}{2}$ $\text{Uhalf} = 4.934802$

Find half-power angles on either side of $\theta_{max} = 0$ by trial and error.

$\theta\text{half} := 0.26017378556$ rad $\frac{\text{Uconical}(\theta\text{half})}{\text{Umax}} = 0.5$

$\text{HPBW} := 2\cdot\theta\text{half}$ $\text{HPBW} = 0.520348$ rad

$\text{HPBWdeg} := \text{HPBW}\cdot\frac{180}{\pi}$ $\boxed{\text{HPBWdeg} = 29.81372}$ deg

b) Use (2-26) to estimate directivity. Note HPBW will be rotationally symmetric for a conical horn.

$\text{Dmax} := \frac{4\cdot\pi}{\text{HPBW}\cdot\text{HPBW}}$ $\boxed{\text{Dmax} = 46.4112}$ $\boxed{10\cdot\log(\text{Dmax}) = 16.666}$ dBi