2.28 The approximate far-zone electric field radiated by a *very thin wire* circular loop of radius a, positioned symmetrically about the z-axis and with its area parallel to the xy-plane, is given by

$$E_\phi \simeq C_0 \sin^{1.5}\theta \frac{e^{jkr}}{r}$$

where C_0 is a constant. Determine the:

(a) Exact directivity (*dimensionless* and *in dB*).

(b) Approximate directivity (*dimensionless* and *in dB*) using an approximate but appropriate formula (*state the formula you are using*).

- For part (a), first find the radiation intensity and normalized radiation intensity. For part (b), first find the HPBW wrt θ.

a) From notes, $W_{rad} = \dfrac{|\bar{E}|^2}{2\eta} = \dfrac{E_\phi E_\phi^*}{2\eta}$

$$W_{rad} = \frac{C_0 \sin^{1.5}\theta \frac{e^{jkr}}{r} C_0^* \sin^{1.5}\theta \frac{e^{-jkr}}{r}}{2\eta} = \frac{C_0^2 \sin^3\theta}{2\eta r^2}$$

Per (2-12), $\underline{\underline{U = r^2 W_{rad} = \dfrac{C_0^2 \sin^3\theta}{2\eta} \quad 0 \le \theta \le 180^0}}$

To normalize, divide by maximum

$$U_{norm} = \frac{U}{U_{max}} = \frac{C_0^2 \sin^3\theta}{2\eta} \Big/ \left(\frac{C_0^2}{2\eta}\right)$$

$$\underline{\underline{U_{norm} = \sin^3\theta \quad 0 \le \theta \le 180^\circ}}$$

For convenience, use U_{norm} to find a P_{rad} and the directivity.

$$(2\text{-}13)\ P_{rad} = \oiint_\Omega U\, d\Omega = \int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi} \sin^4\theta\, d\theta$$

$$= (2\pi - 0)\left[\frac{3\theta}{8} - \frac{\sin 2\theta}{4} + \frac{\sin 4\theta}{32}\right]\Bigg|_{\theta=0}^{\pi}$$

$$P_{rad} = 2\pi\left[\left(\frac{3\pi}{8} - \underbrace{\frac{\sin 2\pi}{4}}_{\to 0} + \underbrace{\frac{\sin 4\pi}{32}}_{\to 0}\right) - \left(0 - \underbrace{\frac{\sin 0}{4}}_{\to 0} + \underbrace{\frac{\sin 0}{32}}_{\to 0}\right)\right]$$

$$= \frac{6\pi^2}{8} = \frac{3\pi^2}{4}$$

(2-16) $D = \frac{4\pi U}{P_{rad}} = \frac{4\pi \sin^3\theta}{3\pi^2/4} \Rightarrow \underline{\underline{D(\theta) = 1.698 \sin^3\theta \quad 0 \le \theta \le 180^\circ}}$

$\underline{\underline{D_{max} = D_0 = 1.698 = 2.2985\ dBi}}$

b) Since the radiation pattern does not change w/ ϕ and is maximum at broadside, i.e., $\theta_{max} = 90^\circ$, use the omnidirectional approx.

$U_{norm} = \sin^3\theta_h = 0.5 \Rightarrow \theta_h = \sin^{-1}(0.5^{1/3})$
$= 52.53269^\circ$ or 127.467°

$HPBW = 127.467 - 52.533 \Longrightarrow \underline{\underline{HPBW = 74.9346^\circ}}$

McDonald (2-33a) $D_0 \simeq \frac{101}{HPBW(deg) - 0.0027[HPBW(deg)]^2}$

$\simeq \frac{101}{74.9346 - 0.0027(74.9346)^2}$

$\underline{\underline{D_0 \simeq 1.6897 = 2.2781\ dBi}}$

Pozar (2-33b) $D_0 \simeq -172.4 + 191\sqrt{0.818 + 1/HPBW(deg)}$

$\simeq -172.4 + 191\sqrt{0.818 + 1/74.9346}$

$\underline{\underline{D_0 \simeq 1.7502 = 2.4309\ dBi}}$

In this case, the McDonald approx. is more accurate.