2.14 Find the directivity (dimensionless and in dB) for the antenna of Problem 2.12 using
(a) Kraus' approximate formula (2-26)
(b) Tai and Pereira's approximate formula (2-30a)

2.12 The normalized radiation intensity of a given antenna is given by (c) $U = \sin\theta \sin^3\phi$
The intensity exists only in the $0 \le \theta \le \pi, 0 \le \phi \le \pi$ region, and it is zero elsewhere. Find the
(a) exact directivity (*dimensionless* and *in dB*).
(b) azimuthal and elevation plane half-power beamwidths (in degrees).

From part b) of problem 2.12c

Azimuthal HPBW $= \Theta_{1D} = 74.9346°$

Elevation HPBW $= \Theta_{2D} = 120°$

a) Per (2-26) + (2-27)

$$D_{max} = D_0 \simeq \frac{4\pi}{\Theta_{1r}\Theta_{2r}} = \frac{4\pi\left(\frac{180}{\pi}\right)^2}{\Theta_{1D}\Theta_{2D}} = \frac{4\pi\left(\frac{180}{\pi}\right)^2}{74.935°(120°)}$$

$$\underline{\underline{D_{max} = D_0 \simeq 4.588 = 10\log_{10} 4.588 = 6.616 \text{ dBi}}}$$

b) Per (2-30a) + (2-30b)

$$D_{max} = D_0 \simeq \frac{32\ln 2}{\Theta_{1r}^2 + \Theta_{2r}^2} = \frac{32\ln 2\left(\frac{180}{\pi}\right)^2}{\Theta_{1D}^2 + \Theta_{2D}^2}$$

$$\simeq \frac{32\ln 2\left(\frac{180}{\pi}\right)^2}{74.935°^2 + 120°^2}$$

$$\underline{\underline{D_{max} = D_0 \simeq 3.638 = 10\log_{10} 3.638 = 5.609 \text{ dBi}}}$$

For comparison, from part a) of 2.12c, $\underline{D_{max} = 6 = 7.78 \text{ dBi}}$