2.12 The normalized radiation intensity of a given antenna is given by (c) $U = \sin\theta \sin^3\phi$
The intensity exists only in the $0 \le \theta \le \pi, 0 \le \phi \le \pi$ region, and it is zero elsewhere. Find the

(a) exact directivity (*dimensionless* and *in dB*).

(b) azimuthal and elevation plane half-power beamwidths (in degrees).

a) (2-16) $D = \dfrac{4\pi U}{P_{rad}}$ and (2-13) $P_{rad} = \oiint U \, d\Omega$

$$P_{rad} = \int_{\phi=0}^{\pi} \int_{\theta=0}^{\pi} (\sin\theta \sin^3\phi) \sin\theta \, d\theta \, d\phi$$

$$= \int_{\phi=0}^{\pi} \sin^3\phi \, d\phi \int_{\theta=0}^{\pi} \sin^2\theta \, d\theta$$

$$= \left[-\cos\phi + \frac{\cos^3\phi}{3}\right]\Bigg|_{\phi=0}^{\pi} \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]\Bigg|_{\theta=0}^{\pi}$$

$$= \left[\left(-(-1)+1\right) + \left(\frac{(-1)^3}{3} - \frac{1^3}{3}\right)\right]\left[\left(\pi/2 - 0\right) - \left(0 - 0\right)\right]$$

$$P_{rad} = (1.\bar{3})(\pi/2) = 2.0943951$$

$$D(\theta, \phi) = \frac{4\pi \sin\theta \sin^3\phi}{2.0943951} = \underline{\underline{6 \sin\theta \sin^3\phi}}$$

@ $\theta = \phi = \pi/2$

$$D_{max} = D_0 = \underline{\underline{6}} = 10 \log_{10} 6 = \underline{\underline{7.7815 \text{ dBi}}}$$

b) <u>Azimuthal HPBW</u> (let $\theta = 90°$)

$$D = 6\underbrace{\sin\theta}_{1}\sin^3\phi_H = \frac{D_0}{2} = 3$$

$$\sin^3\phi_H = 0.5 \Rightarrow \phi_H = \sin^{-1}(0.5)^{1/3}$$

$$= 52.53269° \text{ or } 127.467°$$

$$\text{Azimuthal HPBW} = 2(90° - 52.53269°)$$

or

$$= 127.467° - 52.5327°$$

$$\underline{\underline{\text{Azimuthal HPBW} = 74.935°}}$$

<u>Elevation HPBW</u> (let $\phi = 90°$)

$$D = 6\sin\theta_H \underbrace{\sin^3\phi}_{1} = 3$$

$$\sin\theta_H = 0.5 \Rightarrow \theta_H = \sin^{-1}(0.5)$$

$$= 30° \text{ or } 150°$$

$$\text{Elevation HPBW} = 2(90° - 30°) = 150° - 30°$$

$$\underline{\underline{\text{Elevation HPBW} = 120°}}$$