2.12 The normalized radiation intensity of a given antenna is given by

(b) $U = \sin\theta \sin^2\phi$

The intensity exists only in the $0 \le \theta \le \pi, 0 \le \phi \le \pi$ region, and it is zero elsewhere. Find the

(a) exact directivity (*dimensionless* and *in dB*).

(b) azimuthal and elevation plane half-power beamwidths (in degrees).

(a) $D_{max} = D_o = \dfrac{4\pi U_{max}}{P_{rad}}$ (2-16a)

$U_{max} = 1$ (occurs when $\theta = \pi/2$ + $\phi = \pi/2$)

$$P_{rad} = \oiint_{\Omega} U \, d\Omega = \oint\oint U \sin\theta \, d\theta \, d\phi \quad (2\text{-}13)$$

$$= \int_{\theta=0}^{\pi} \sin^2\theta \, d\theta \int_{\phi=0}^{\pi} \sin^2\phi \, d\phi$$

$$= \left(\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right)\Big|_0^{\pi} \left(\frac{\phi}{2} - \frac{\sin 2\phi}{4}\right)\Big|_0^{\pi}$$

$$= (\pi/2)^2$$

$$D_{max} = \frac{4\pi(1)}{(\pi/2)^2} = 5.093 = 7.07 \text{ dBi}$$

(b) for $\phi = \pi/2$, $U = \sin\theta = 0.5$ when $\theta = 30°$ $U_{max}(\theta = 90°)$

$HPBW_{elevation} = 2(90° - 30°) = 120°$

for $\theta = \pi/2$, $U = \sin^2\phi = 0.5$ when $\phi = 45°$ $U_{max}(\phi = 90°)$

$HPBW_{azimuthal} = 2(90° - 45°) = 90°$