

**2.10** In target-search ground-mapping radars it is desirable to have echo power received from a target, of constant cross section, to be independent of its range. For one such application, the desirable radiation intensity of the antenna is given by

$$U(\theta, \phi) = \begin{cases} 1 & 0^\circ \leq \theta < 20^\circ \\ 0.342 \csc(\theta) & 20^\circ \leq \theta < 60^\circ \\ 0 & 60^\circ \leq \theta \leq 180^\circ \end{cases} \quad 0^\circ \leq \phi \leq 360^\circ$$

Find the directivity (in dB) using the exact formula.

$$(2-16) \quad D = \frac{4\pi U}{P_{rad}}$$

$$\begin{aligned} (2-13) \quad P_{rad} &= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} U \sin\theta \, d\theta \, d\phi = \int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi} U \sin\theta \, d\theta \\ &= 2\pi \left[ \int_{\theta=0}^{20^\circ} (1) \sin\theta \, d\theta + \int_{\theta=20^\circ}^{60^\circ} 0.342 \csc(\theta) \sin\theta \, d\theta + 0 \right] \\ &= 2\pi \left[ -\cos\theta \Big|_0^{20^\circ} + 0.342 \theta \Big|_{20^\circ}^{60^\circ} \right] \\ &= 2\pi \left[ -\cos 20^\circ + \cos 0 + 0.342 (60^\circ - 20^\circ) \right] = 1.8791 \text{ W} \end{aligned}$$

$$D(\theta, \phi) = \begin{cases} \frac{4\pi(1)}{1.8791} & 0^\circ \leq \theta < 20^\circ \\ \frac{4\pi(0.342 \csc(\theta))}{1.8791} & 20^\circ \leq \theta < 60^\circ \\ \frac{4\pi(0)}{1.8791} & 60^\circ \leq \theta \leq 180^\circ \end{cases}$$

$$D(\theta) = \begin{cases} 6.687433 & 0^\circ \leq \theta < 20^\circ \\ 2.2871 \csc(\theta) & 20^\circ \leq \theta < 60^\circ \\ 0 & 60^\circ \leq \theta \leq 180^\circ \end{cases}$$

$$D_0 = D_{max} = 6.687433 = 10 \log_{10} 6.687433$$

$$\underline{\underline{D_0 = 8.2526 \text{ dB}}}$$