2.8 The *approximate far zone normalized electric field* radiated by a *resonant* linear dipole antenna used in wireless mobile units, positioned symmetrically at the origin along the z-axis, is given by

$$\mathbf{E}_a \simeq \hat{\mathbf{a}}_\theta E_a \sin^{1.5}\theta \frac{e^{-jkr}}{r}, \qquad \begin{matrix} 0° \le \theta \le 180° \\ 0° \le \phi \le 360° \end{matrix}$$

where E_a is a constant and r is the spherical radial distance measured from the origin of the coordinate system. Determine the:

(a) *Exact maximum directivity* (*dimensionless* and *in dB*).

(b) Half-power beamwidth (*in degrees*)

(c) *Approximate maximum directivity* (*dimensionless* and *in dB*). Indicate which approximate formula you are using and why?

(d) *Approximate maximum directivity* (*dimensionless* and *in dB*) using another approximate formula. Indicate which other formula you are using and why?

(e) Maximum directivity (dimensionless and in dB) using the computer program **Directivity**.

- First, find the radiation intensity and normalized radiation intensity.

From notes, $W_{rad} = \frac{|\bar{E}|^2}{2\eta} = \frac{\bar{E}_a \cdot \bar{E}_a^*}{2\eta}$

$$W_{rad} = \frac{1}{2\eta} \hat{a}_\theta E_a \sin^{1.5}\theta \frac{e^{-jkr}}{r} \cdot \hat{a}_\theta E_a \sin^{1.5}\theta \frac{e^{+jkr}}{r}$$

$$= \frac{E_a^2 \sin^3\theta}{2\eta r^2}$$

Per (2-12), $U = r^2 W_{rad} = \frac{E_a^2 \sin^3\theta}{2\eta}$ $\quad 0 \le \theta \le 180°$, $0 \le \phi \le 360°$

To normalize, divide by maximum U_{max}

$$U_{norm} = \frac{E_a^2 \sin^3\theta}{2\eta} \Big/ \left(\frac{E_a^2}{2\eta}\right) \Rightarrow \sin^3\theta = U_{norm} \quad 0 \le \theta \le 180°,\ 0 \le \phi \le 360°$$

a) For convenience, use U_{norm} to find P_{rad} & directivity.

(2-13) $P_{rad} = \oiint_\Omega U \, d\Omega = \int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi} \sin^4\theta \, d\theta$

$$= (2\pi - 0)\left[\frac{3\theta}{8} - \frac{\sin 2\theta}{4} + \frac{\sin 4\theta}{32}\right]\Bigg|_{\theta=0}^{\pi} = 2\pi\left(\frac{3\pi}{8}\right)$$

$$= \frac{3\pi^2}{4}$$

(2-16) $D = \frac{4\pi U}{P_{rad}} = \frac{4\pi \sin^3\theta}{3\pi^2/4} \Rightarrow D = 1.698 \sin^3\theta$ $\quad 0 \le \theta \le 180°$, $0 \le \phi \le 360°$

$$D_{max} = D_0 = 1.698 = 2.2985 \text{ dBi}$$

b) $U_{norm} = \sin^3\theta_h = 0.5 \Rightarrow \theta_h = \sin^{-1}(0.5^{1/3})$

$$= 52.5327° \text{ or } 127.467°$$

$$HPBW = 127.467° - 55.533° \Rightarrow HPBW = 74.9346°$$

c) Since $\theta_{max} = 90°$ is @ broadside & D does not change w/ ϕ, use omnidirectional approx.

McDonald (2-33a) $D_0 \simeq \frac{101}{HPBW(deg) - 0.0027[HPBW(deg)]^2}$

$$\simeq \frac{101}{74.9346 - 0.0027(74.9346)^2}$$

$$D_0 \simeq 1.6897 = 2.2781 \text{ dBi}$$

d) Pozar (2-33b) $D_0 \simeq -172.4 + 191\sqrt{0.818 + 1/HPBW(deg)}$

$$\simeq -172.4 + 191\sqrt{0.818 + 1/74.9346}$$

$$D_0 \simeq 1.7502 = 2.4309 \text{ dBi}$$

In this case, the McDonald approx. is more accurate.

e)

Define U.m

```
% m-file U.m
function y = U(THETA, PHI)
y = (sin(THETA)).^3;
```

Call DIRECTIVITY.m from Matlab Command Window

>> DIRECTIVITY

!!!WARNING: Make sure you define radiation intensity in file U.m !!!

Output device option

Option (1): Screen

Option (2): File

Output device=1

The lower bound of theta in degrees = 0

The upper bound of theta in degrees = 180

The lower bound of phi in degrees = 0

The upper bound of phi in degrees = 360

Input parameters:

The lower bound of theta in degrees = 0

The upper bound of theta in degrees = 180

The lower bound of phi in degrees = 0

The upper bound of phi in degrees = 360

Output parameters:

Radiated power (watts) = 7.40426

Directivity (dimensionless) = 1.69718

Directivity (dB) = 2.29728