

- 2.7 The power radiated by a lossless antenna is 10 watts. The directional characteristics of the antenna are represented by the radiation intensity of

$$U = B_0 \cos^5 \theta \quad (\text{watts/unit solid angle}) \quad (0 \leq \theta \leq \pi/2, 0 \leq \phi \leq 2\pi)$$

For each, find the

- maximum power density (in watts/square meter) at a distance of 1,000 m (assume far-field distance). Specify the angle where this occurs.
- exact and approximate beam solid angle Ω_A .
- directivity, exact and approximate, of the antenna (dimensionless and in dB).
- gain, exact and approximate, of the antenna (dimensionless and in dB).

a) Per (2-12), $U = r^2 W_{\text{rad}} \Rightarrow W_{\text{rad}} = \frac{U}{r^2}$

Since $r = 1000 \text{ m}$ is fixed, the maximum in W_{rad} occurs when $U = U_{\text{max}} = B_0 \cos^5 0 = B_0 @ \theta_{\text{max}} = 0$.

To determine the constant B_0 , use eq'n (2-13)

$$P_{\text{rad}} = \oint_{\Omega} U d\Omega \Rightarrow 10 \text{ W} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} B_0 \cos^5 \theta \sin \theta d\theta d\phi$$

$$10 = B_0 \int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi/2} \cos^5 \theta \sin \theta d\theta = B_0 (\phi) \Big|_0^{2\pi} \left(\frac{-\cos^6 \theta}{6} \right) \Big|_0^{\pi/2}$$

$$10 = B_0 (2\pi - 0) \left[\frac{-\cos^6 \theta}{6} \Big|_0^{\pi/2} + \frac{\cos^6 0}{6} \right] = B_0 (2\pi/6)$$

$$\hookrightarrow B_0 = \frac{30}{\pi}$$

$$W_{\text{rad, max}} = \left. \frac{U_{\text{max}}}{r^2} \right|_{r=1000 \text{ m}} = \left. \frac{B_0}{r^2} \right|_{r=1000 \text{ m}} = \frac{30/\pi}{1000^2}$$

$$W_{\text{rad, max}} = 9.5493 \times 10^{-6} \text{ W/m}^2 = 9.5493 \frac{\mu\text{W}}{\text{m}^2}$$

$@ \theta_{\text{max}} = 0$

b) Per (2-19), $U = B_0 F(\theta, \phi) \Rightarrow B_0 \cos^5 \theta = B_0 F(\theta, \phi)$

$$\hookrightarrow F(\theta, \phi) = \cos^5 \theta \quad (0 \leq \theta \leq \pi/2)$$

b) cont. Per (2-24), the exact beam solid angle is

$$\begin{aligned}\Omega_A &= \frac{1}{F(\theta, \phi)|_{\max}} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} F(\theta, \phi) \sin\theta \, d\theta \, d\phi \\ &= \frac{1}{(\cos^5\theta)|_{\max}} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \cos^5\theta \sin\theta \, d\theta \, d\phi \quad \text{where } \cos^5\theta|_{\max}=1 \\ &\quad \text{@ } \theta_{\max}=0 \\ &= \frac{1}{1} \int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi/2} \cos^5\theta \sin\theta \, d\theta = 2\pi (1/6)\end{aligned}$$

$$\underline{\underline{\Omega_A = \pi/3 \text{ Sr} = 1.0472 \text{ Sr}}}$$

Per (2-26a), the approximate beam solid angle is

$$\Omega_A \approx \theta_{1r} \theta_{2r} \quad \text{where } \theta_{1r} \text{ \& } \theta_{2r} \text{ are HPBW's}$$

$$U(\theta_{hp}) = \frac{b_0}{2} = b_0 \cos^5\theta_{hp} \Rightarrow \cos\theta_{hp} = (0.5)^{1/5}$$

$$\theta_{hp} = \cos^{-1}(0.5)^{1/5} = 29.477^\circ = 0.51448 \text{ rad}$$

$$\theta_{1r} = \theta_{2r} = 2\theta_{hp} = 1.029 \text{ rad}$$

$$\underline{\underline{\text{Approx. } \Omega_A \approx 1.029^2 = 1.0587 \text{ Sr}}}$$

c) Per (2-21), $D(\theta, \phi) = D(\theta) = \frac{4\pi F(\theta, \phi)}{\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} F(\theta, \phi) \sin\theta \, d\theta \, d\phi}$

$$\underline{\underline{D(\theta) = \frac{4\pi \cos^5\theta}{2\pi (1/6)} = 12 \cos^5\theta \quad (0 \leq \theta \leq \pi/2, 0 \leq \phi \leq 2\pi)}}$$

$$(2-23) \underline{\underline{D_{\max} = D_0 = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\pi/3} = 12 = 10.79 \text{ dBi}}}$$

$$(2-26) \underline{\underline{D_{\max} = D_0 \approx \frac{4\pi}{\Omega_{A, \text{approx}}} = \frac{4\pi}{1.0587} = 11.869 = 10.744 \text{ dBi}}}$$

d) Lossless antenna $\Rightarrow \underline{\underline{G = D}}$ (same answers!)